A STOCHASTIC OPTIMAL CONTROL FORMULATION OF THE RISK BALANCING DEBT CHOICE MODEL: A BASIS FOR GENERALIZED METHOD OF MOMENTS ESTIMATION OF RISK AVERSION COEFFICIENTS

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The solution of stochastic optimal control problems, under the assumptions of constant relative and constant absolute risk aversion, yields two sets of optimal paths for the debt-to-asset ratio and consumption over time. Analysis of these optimal paths reveals that important characteristics of individual behavior depend on whether constant relative or constant absolute risk aversion is assumed. For example, under constant relative risk aversion, the optimal leverage position does not depend on the firm's equity holdings, while under constant absolute risk aversion, the optimal leverage position decreases as equity increases. Furthermore, the optimal average propensity to consume (out of disposable income) is also independent of equity value under constant relative risk aversion. Under constant absolute risk

aversion, however, as the firm accumulates equity over time, the optimal average propensity to withdraw income from the farm operation decreases.

The second part of the study deals with estimating risk aversion measures and testing the restrictions implied by the optimal path equations under the assumptions of constant absolute and constant relative risk aversion, for the aggregate agricultural sector. Even though the restrictions implied by the assumption of constant absolute risk aversion cannot be rejected using the only (asymptotic) test available, the study concludes that the associated optimal paths do not describe the behavior of the aggregate agricultural sector very well. In addition, since the risk aversion measure cannot be placed within empirically meaningful bounds under constant absolute risk aversion, using the estimated optimal paths in simulation or comparative statics would lack any credibility from a statistical standpoint.

The optimal paths associated with the assumption of constant relative risk aversion, on the other hand, appear to be quite consistent with the aggregate behavior of the agricultural sector. Furthermore, the risk aversion measure can be placed within empirically meaningful bounds in this case. Results from comparative statics analysis investigating the effect of changes in the exogenous paths on the optimal leverage position and consumption prove to be quite reliable, from a statistical perspective.

CHAPTER 1
INTRODUCTION

Agriculture in the United States suffered increased financial stress during the 1980s. Large debt loads incurred during the more prosperous 1970s turned to financial burdens as output and real estate prices declined. As a result, the agricultural sector underwent a period of foreclosures at the farm level and financial difficulties among agricultural intermediaries. The "debt is good" attitude of the 1970s was replaced by a more cautious approach.

The difficulties arising from the debt situation of the 1980s have spurred increased research into the appropriate level of farm debt and the factors influencing the farmer's leverage decision. However, as Collins (1985a) points out, the area within the theory of the agricultural firm dealing with the optimal debt-equity structure is still to be adequately developed. The first development of a theory of capital structure in economics is within the context of publicly held corporations (Modigliani and Miller, 1966). Unfortunately, the Modigliani-Miller arguments do not apply for the agricultural sector where production is dominated by proprietorships and nonpublicly held corporations. The arbitrage forces developed in the equity markets for publicly

held corporations when they are over or under leveraged do not exist in the case of agriculture.

More recently, Titman and Wessels (1988) analyzed the explanatory power of some of the contemporary theories of optimal capital structure. These theories, developed by researchers within the area of finance, also deal with economic entities quite different from the typical agricultural firm and assume market forces of a distinct nature.

Research efforts aimed to improve the understanding of the factors that determine the leverage position of the proprietary firm typical of the agricultural sector could be useful in several policy areas. A validated theory of capital structure could be useful in constructing structural models of the agricultural credit markets. It would also provide an important tool to evaluate both the response of individual firms and the aggregate agricultural sector to policy changes affecting farm income and risk.

One of the most popular theoretical works dealing with the development of such a theory has been the model advanced by Collins (1985b). This model has been used by Featherstone et al. to examine the effect of government programs on the leverage position of the agricultural firm and the probability of equity loss. Moss et al. (in press) applied a similar version of this model to evaluate the effect of changes in tax code on leverage and the probability of equity loss. Turvey and Baker (1989) and Moss and van Blokland (1989) have used

Collins' basic formulation to study the interaction between the choice of debt and the choice of marketing instruments.

Collins' model has probably gained acceptance because of its simplicity. However, its basic strength may also be its greatest weakness. One of the simplifications in Collins' model arises from collapsing the decision-making process into a single period. Collins himself suggests further research in continuous time formulations, specifically within the stochastic optimal control framework (Collins, 1985a). In addition, Collins' formulation is based on maximizing expected utility of the rate of return on equity and makes no explicit consideration of the consumption-investment decision also faced by the entrepreneur. This study attempts to rectify these weaknesses by reformulating Collins' risk-balancing model into a more complete and comprehensive stochastic optimal control framework.

A second undertaking of this study is the development of a theoretical framework for the econometric estimation of risk aversion coefficients within the context of agricultural production based on the hypotheses that emerge from the solution of the stochastic optimal control problem. Different techniques for estimating risk aversion coefficients have been proposed in the literature. The most direct procedure requires the derivation of the decision maker's utility function which translates revealed choices to scalar indices of desirability. This technique is based on preference

elicitation interviews which, as Binswanger (1980) points out, yield information of questionable value due to the shortcomings in the design of such interviews. In addition, the estimation procedures for the elicited utility function are plagued with statistical difficulties.

King and Robison (1981), recognizing the shortcomings of the available techniques to estimate risk aversion coefficients, proposed a method for constructing interval measurements of decision makers' absolute risk aversion functions. These interval measurements can be used in conjunction with the criteria of stochastic dominance with respect to a function to order uncertain choices.

Antle (1987) proposed a general method for the identification and econometric estimation of the parameters of the distribution of risk attitudes in a producer population. His structural econometric production model consists of the conditional distribution function of profits given variable and fixed input quantities to be used in the productive process and the first-order conditions for maximization of expected utility of profits from a single-period optimization model.

The approach applied in this study involves obtaining an estimable system of relationships, which contain risk aversion coefficients as structural parameters, is along the lines of Antle's. Our framework, however, is one of stochastic multiperiod optimization. It accounts for the random nature of

economic and financial events in an unrestricted fashion. Our relationships are derived from financial considerations resulting from the leverage decision, such that the only unknown parameters in the system are risk aversion coefficients for the different agents under consideration.

Once a system of estimable relationships has been defined, there are basically two strategies for obtaining econometric estimates of the structural parameters in such relationships. One approach is to explicitly specify the complete economic environment including the stochastic properties of the forcing variables, solve for an equilibrium representation of the endogenous variables in terms of past endogenous and current and past forcing variables, and estimate the unknown parameters and the stochastic process governing the forcing variables using a maximum likelihood procedure. If a full information maximum likelihood procedure is used, the estimators will, in general, be asymptotically more efficient than those obtained by nonlinear instrumental variables techniques if the distributional assumptions regarding the forcing variables are specified correctly. On the other hand, maximum likelihood estimators may fail to be consistent if the distributions are misspecified. Thus, there exists a tradeoff between robustness and computational simplicity versus efficiency with a possibility of inconsistency. In addition, properly implementing full information maximum likelihood estimation within this context is typically a very

complex task. The complexity arises when attempting to specify the proper likelihood function needed to implement a full information maximum likelihood procedure, since some of the random forcing variables may enter nonlinearly in some of the optimal path equations. Some sort of limited information procedure using only the optimal paths with simpler stochastic characteristics may be the only implementable estimation technique within the class of maximum likelihood in this case.

The second approach is the generalized method of moments (GMM) instrumental variables procedure set out by Hansen (1982). The procedure has been used in several financial market applications (Brown and Gibbons 1985; Dunn and Singleton 1984; Rotenberg 1984) and in labor market applications as well (Biddle 1984). The basic idea behind this approach is to apply the GMM estimator directly to orthogonality conditions implied by the first order conditions of the agents' intertemporal optimization problems. Specifically, residuals are formed by using realized values when conditional expectations appear in the first order conditions, and the instruments are simply variables known at the time the expectation is formed. The procedure is a limited-information method analogous to two-stage least squares. Among its attractive features are that it does not require strong distributional assumptions nor a complete description of the agents' environment. As noted by Garber and King (1983), however, the procedure does entail the implicit assumption that the first-order conditions are

exact in the sense that the functional form of the agents' objective function is known to the econometrician and not subject to unobserved random fluctuations. Furthermore, the GMM approach does not incorporate nearly as wide a class of specification tests as those developed by Newey (1985) and Tauchen (1985) for parametric maximum likelihood models. Some other disadvantages of the GMM procedure are the fact that there is little guidance from econometric theory for choosing the appropriate lag lengths for forming the instruments, and that all of the econometric theory justifying inference from GMM estimates is asymptotic. For sample sizes such as the one available in this study, there is little information regarding the properties of the estimates and the quality of the asymptotic approximations, as well as the validity the size of the one specification test available. There is no evidence regarding the power of the test of the overidentifying restrictions implied by the first order conditions of the dynamic optimization problem.

Despite the aforementioned disadvantages associated with the GMM approach it is my judgement that, given the nature and purpose of this study, GMM is the most appropriate strategy to follow.

Objectives

The objectives of this study are:

(1) To formulate and solve multi-period, dynamic, stochastic optimal control models for the nonpublicly held firm's optimal withdrawal-reinvestment and leverage decisions under the assumptions of constant absolute and constant relative risk aversion. The solutions for these models should provide insights regarding the determinants of endogenous control variables such as the consumption level and debt-to-asset ratio as well as endogenous state variables such as equity. Furthermore, they should yield the precise functional forms of the optimal relationships among the endogenous variables, given the exogenous paths (expected rate of return to assets, variance of the rate of return to assets, cost of debt capital and discount rate) and the unknown parameters (risk aversion coefficients).

(2) To investigate the empirical implications of assuming constant absolute versus constant relative risk aversion regarding both the optimal leverage position and the optimal consumption path over time.

(3) To explore the possibilities for estimating risk aversion coefficients based on the optimality relationships obtained from the solution procedures and the solutions themselves of the stochastic optimal

control problems applying the generalized method of moments technique. The generality of the assumptions required to specify and solve the theoretical models and to implement the GMM technique, as well as the soundness of the financial relationships on which the theoretical models are built (there is only one unknown parameter to be estimated--the risk aversion coefficient), are considered very attractive characteristics of the proposed approach.

(4) To try to ascertain whether the behavior of the aggregate agricultural sector is more likely to be consistent with constant absolute or constant relative risk aversion.

Data

In order to estimate scalar measures of the inherent level of risk associated with the aggregate agricultural sector and test the different hypotheses regarding the risk behavior and the nature of the relationships characterizing the evolution of the capital structure of the sector implied by the constant absolute and constant relative risk aversion assumptions, the following data are required:

- Debt-to-asset ratio.
- Total rate of return to assets.
- Variance of the total rate of return to assets.
- Cost of debt capital.

- Income withdrawal.
- Equity value.
- Inflation rate.

These data were obtained from two United States government publications: The National Financial Summary of 1988 by the United States Department of Agriculture and the Agricultural Finance Databook by the Division of Research and Statistics of the Board of Governors of the Federal Reserve System.

CHAPTER 2
METHODOLOGY

The Theory of Stochastic Optimal Control

One of the most common uses of differential calculus is the optimization of functions of one or more variables. These variables may be constrained to obey one or more equations or inequalities. The calculus of variations applies to problems of a similar nature--it considers magnitudes, not functions, that depend not on the values of variables but on functions. The magnitudes, which are indeed functions of functions, are called functionals. The solution of the calculus of variations problem looks for a function so as to maximize (or minimize) a functional.

A generalization of the classical calculus of variations is the **maximum principle** for optimal control, developed in the late 1950s by Pontryagin et al (1962). In optimal control problems, variables are divided into two types--**state** variables and **control** variables. The movement of state variables is governed by first-order differential equations.

In a more sophisticated formulation of the optimal control problem, the movement of the state variable is subject to a stochastic disturbance. Instead of the usual differential equation governing the movement of the state variable, a formal stochastic differential equation is introduced:

$$dx(t) = g(t,x(t),u(t))dt + f(t,x(t),u(t))dz(t) \tag{2.1}$$

where g and f are assumed to be known and continuously differentiable functions of the independent arguments t, x(t), and u(t), none of which is a derivative. The control variables, represented by the vector u(t), and the state variable x(t), are piecewise continuous functions of time to be determined by the solution of the stochastic optimal control problem. The term dz(t) is the increment of a stochastic process, z(t), that obeys what is called **Brownian motion** or **white noise.** The stochastic process, z(t), is also known as a **Wiener** process. Notice that the expected rate of change of x(t), E [dx(t)/dt] is g, but there is a disturbance term, an uncertain component of the rate of change. Briefly, for a Wiener process z(t), and for any partition t_0, t_1, t_2, . . . of the time interval, the random variables $z(t_1)-z(t_0)$, $z(t_2)-z(t_1)$, $z(t_3)-z(t_2)$, . . . are independently and normally distributed with mean zero and variances t_1-t_0, t_2-t_1, t_3-t_2, . . ., respectively.

The general form of the stochastic optimal control problem to be solved in this study is the following:

$$\text{MAX } E\left[\int_0^{\infty} e^{-rt} \; F(x(t),u(t))dt\right] \tag{2.2}$$

subject to $x(t_0) = x_0$, and

$dx(t) = g(x(t),u(t))dt + f(x(t),u(t))dz(t)$;

where $F(x(t),u(t))$ is the functional to be maximized.

The basic equation for a problem of this nature is (Kamien and Schwartz):

$$(2.3) \qquad rV(x(t)) = \max_{u}\left[F(x(t),u(t)) + V'(x(t))g(x(t),u(t)) + .5f^2(x(t),u(t))V''(x(t)) \right].$$

where $V(x(t))$ is an unknown function of the state variable. Maximizing the right-hand side of (2.3) with respect to the control variables, and substituting in the optimal value functionals, yields a nonlinear second-order differential equation. The solution to this equation (i.e., the functional form of $V(x(t))$) is all that is needed to determine the optimal paths for the control variables which are indeed the optimal value functionals evaluated at the solution for $V(x(t))$.

Theory Underlying the Model Specification

Theoretically, it is assumed that the farmer chooses actions to maximize expected utility through time. The level of risk inherent to the different available alternatives can be an important consideration within this framework. Historical research into risk management was primarily concerned with mechanisms which control the level of business risk. Barry and Robison define business risk as the risk associated with the firm's asset composition. Hence, early studies mainly dealt with decisions such as the choice of marketing strategies and enterprise mix to control such risks as output price

volatility and weather. Gabriel and Baker (1980) shifted the focus to the control of financial risk, by adding debt level to the list of variables at the firm's disposal to control risk. Financial risk results from both leveraging decisions and business risk. If the business risk of a particular group of agricultural assets is high, the producer may wish to reduce total risk by reducing the level of debt.

The interrelationships between the different variables characterizing a firm's financial and economic situation can be developed starting from the duPont identity (Collins, 1985b):

$$R_p / E = (R_p / A) (A / E). \tag{2.4}$$

where R_p is the net expected return to the portfolio of enterprises, A is the value of assets held by the firm and E is the value of its equity. The ratio of assets to equity is a leverage measure that may be thought of as the number of dollars of assets that are supported by one dollar of equity. Equation (2.4) can also be written as

$$R_p / E = (R_p / A) (1 / (1 - \delta)). \tag{2.5}$$

where δ is the ratio of debt to asset value. As Collins points out, however, equation (2.5) ignores two important factors in the leverage choice decision: the interest rate associated with the debt and anticipated increases in asset values. With an interest rate of K and a debt value of D, the

effect of debt in the rate of return to assets is $-KD/A$ or $-K\delta$. If i is defined to be the anticipated rate of increase in asset value, the effect of asset valuation on the expected rate of return to assets is iA/A, or i itself. Therefore, the net rate of return to equity can be expressed as

$$(2.6) \qquad R_e = [\ (\ R_p\ /\ A\ ) + i - (\ K\ \delta\ )\ ]\ \ /\ (\ 1 - \delta\ )$$

In addition, a gross anticipated rate of return to assets can be defined to be $R_A = (R_p/A)+i$; so that equation (2.6) can also be written as

$$(2.7) \qquad R_e = [\ R_A - \ (\ k\ \delta\ )\ ]\ /\ (\ 1 - \delta\ )$$

Equation (2.7) and the random nature of R_A are the basis for Collins' model. In his formulation, leverage is the only choice variable that the decision maker can use to maximize expected well being because it collapses the decision-making process to a single period. The relationship is based on maximizing expected utility of the rate of return on equity and cannot explicitly address the related consumption investment decision also faced by the entrepreneur. The stochastic optimal control formulation proposed in this study is also based on equation (2.7) and the random nature of the gross rate of return to assets, but it explicitly integrates the consumption-investment decision into an intertemporal dynamic specification of the debt choice model. The decision maker maximizes a discounted stream of expected utility (derived

from consumption) by simultaneously making a reinvestment-withdrawal decision and selecting the optimal level of leverage each time period.

The Stochastic Optimal Control Formulation

Starting from equation (2.7), which specifies the net rate of return to equity Re, the change in equity value in any time period t can be defined as

$$\text{(2.8)} \qquad dE(t) = \{E(t)[R_A(t)-K(t)\delta(t)]/(1-\delta(t))- C(t)\}dt$$

where E(t) is the equity value of the proprietor and C(t) is the level of consumption (withdrawal from the firm) at time period t. The total rate of return to assets, cost of debt capital and leverage ratio are now expressed as continuous functions of time. Notice that the total rate of return on farm assets and cost of debt capital are allowed to vary over time, but they are not controllable by the decision maker; i.e., they follow exogenously determined paths.

The stochastic nature of the problem arises from the fact that the total rate of return to assets, which depends on such things as product yields and prices as well as asset valuation rates, is not known with certainty when the relevant withdrawal-reinvestment and leverage decisions are made. It is assumed, however, that the decision maker knows the expected value and variance of this random variable through time. Therefore equation (2.8) has as its stochastic counterpart

$$(2.9) \qquad dE(t) = \{E(t)[\hat{R}_A(t)-K(t)\delta(t)]/(1-\delta(t))-C(t)\}dt + \{E(t)\sigma_A(t)/(1-\delta(t))\}dz(t)$$

where $\hat{R}_A(t)$ is the anticipated or expected rate of return to assets, $\sigma_A(t)$ is the standard deviation of R_A, and $dz(t)$ is the increment of stochastic process that obeys "Brownian motion" or "white noise." Thus, $dz(t)$ is the limit of a Gaussian random variable.

Equation (2.9) defines infinitesimal changes in equity over time to be the infinitesimal expected changes in equity over time plus an infinitesimal error term. This error term is proportional to a standard normal disturbance, with the proportionality factor depending on the standard deviation of the equity differential. Therefore, this stochastic specification implicitly assumes that the forcing variable $Ra(t)$ is normally distributed for time periods of any length.

The entrepreneur's optimization problem is to maximize the expected present value of an infinite stream of utility given a discount rate r:

$$(2.10) \qquad \text{Max } E \left(\int_0^\infty e^{-rt} F[C(t),P]dt \right)$$

subject to the equation of motion (2.9) and an initial level of equity $E(0) = E_0$. Specifically, $F[C(t),P] = (-e^{-\lambda C(t)})/\lambda$, $\lambda > 0$, under constant absolute risk aversion and $F[C(t),P] = (C(t)^b)/b$, $b < 1$, under constant relative risk aversion.

There are three different types of variables entering the stochastic optimal control problem defined by equation (2.9) and (2.10): exogenous paths, endogenous control variables, and endogenous state variables. The exogenous paths are variables that can continuously change over time but that the decision maker cannot control. They are the expected total rate of return to assets, variance of the total rate of return to assets, cost of debt capital, and the discount rate. The solution of the stochastic optimal control problem yields optimal paths for the endogenous control variables which can be instantaneously adjusted by the entrepreneur in response to changes in the exogenous paths. The endogenous control variables are the leverage position (i.e., debt-to-asset ratio) and the consumption rate. The optimal paths followed by the endogenous control variables determine the optimal levels of the endogenous state variables over time. The endogenous state variables are equity and asset-value holdings.

The optimality conditions are expected to offer important insights into the relationships between endogenous paths such as equity value accumulation and the rate of withdrawals (consumption) and reinvestment over time and exogenous paths such as the expected rate of returns to assets, the variance of the rate of return to assets (also referred to as business risk) and the cost of debt capital for the non-publicly held agricultural firm. Analyzing the effect of changes in the

exogenous paths on the probability distributions of the endogenous variables at given future time periods based on the optimality conditions is possible.

Econometric Estimation of Risk Aversion Coefficients

Econometric estimation of the risk aversion coefficients will be attempted by applying the **generalized method of moments** technique.

The generalized method of moments technique is an econometric estimation strategy that circumvents the theoretical requirement of an explicit representation of the stochastic equilibrium, yet permits identification and estimation of parameters of economic agents' dynamic objective functions, as well as tests of the overidentifying restrictions implied by the theoretical model (Hansen et al., 1982). The basic idea underlying this estimation strategy is that the dynamic optimization problems of economic agents typically imply a set of **stochastic Euler equations** that must be satisfied in equilibrium. These Euler equations, in turn, imply a set of population orthogonality conditions that depend, in a nonlinear way, on variables observed by an econometrician and on unknown parameters characterizing preferences, profit functions, etc. Hansen and Singleton construct **nonlinear instrumental variables estimators** for these parameters in the manner suggested by Amemiya (1974) and Jorgenson and Laffont (1974) by making sample versions of the orthogonality conditions close to zero according to a certain metric. There are two

important features regarding the generalized method of moments estimators. First, under fairly weak assumptions about the random processes generating the observable data, they are consistent and have a limiting normal distribution. In addition, it is generally the case that more orthogonality conditions are available than parameters to be estimated so that, in a sense, the models are overidentified. The overidentifying restrictions can be tested using a procedure that examines how close to zero sample versions of population orthogonality conditions are (Hansen, 1982).

Turning now to the specifics of the estimation procedure, consider the set of first-order conditions (stochastic Euler equations) from a dynamic optimization problem:

$$E_t \left[h \left(Y_{t+n} , B \right) \right] = 0. \tag{2.11}$$

where Y_{t+n} is a k dimensional vector of variables observed by agents and the econometrician as of date t+n, B is an l dimensional parameter vector that is unknown to the econometrician, h is a function mapping $R^k \times R^l$ into R^m, and E_t is the expectations operator conditioned on agents' period t information set, I_t.

Assume that the m constituents of h have finite second moments. Also, let Z_t denote a q dimensional vector of variables with finite second moments that belong to I_t and are observed by the econometrician, and define the function F by

$$F \left(Y_{t+n} , Z_t , b \right) = h \left(Y_{t+n} , b \right) .*. Z_t. \tag{2.12}$$

where F maps $R^k \times R^q \times R^l$ into R^r, $r = m \times q$, and .*. is the Kronecker product operator.

Then an implication of (2.11), (2.12), and the preceding assumptions is that

$$(2.13) \qquad E [F (Y_{t+n} , Z_t , B)] = 0.$$

where E is the **unconditional** expectations operator. Equation (2.13) provides a set of r population orthogonality conditions. From these conditions, an estimator for B can be derived if r is greater than or equal to l, the number of parameters to be estimated.

It is now necessary to construct an objective function that depends only on the available sample information $\{ (Y_{1+n}, Z_1) , (Y_{2+n} , Z_2) , ... , (Y_{T+n} , Z_T) \}$ and the unknown parameters. For this purpose, define

$$(2.14) \qquad G_0 (b) = E [F (Y_{t+n} , Z_t , b)].$$

where b belongs to R^l and the function G_0 is the same for all time periods. Note that, by (2.13), if b = B, G_0 must take the value of zero.

It then follows that, if the theoretical model from which (2.11) was derived is an adequate representation of reality, the method of moments estimator of the function G_0 is

$$(2.15) \qquad G_T (b) = (1 / T) \sum_{t=1}^{T} (F (Y_{t+n} ,Z_t , b)).$$

From the preceding arguments, it is clear that $G_T(b)$ should be close to zero when evaluated at $b = B$ for large samples. If F is continuous in its third argument, it is then reasonable to estimate B by selecting a b from a subset of R^l that makes $G_T(b)$ "close" to zero. Following Amemiya (1977), Jorgenson and Laffont (1974), and Hansen (1982), b can be chosen to minimize the function J_T given by

$$(2.16) \qquad J_T(b) = G_T(b)' W_T G_T(b).$$

where W_T is an r by r symmetric, positive definite matrix that can depend on sample information. The choice of the weighting matrix W_T defines the metric used in making G_T close to zero.

The sufficient conditions for strong consistency and asymptotic normality of method of moments estimators constructed in this fashion are given by Hansen (1982). In addition, some important observations about the technique are in order:

First, there is no need to specify how $E_t[h(Y_{t+n}, B) h(Y_{t+n}, B)']$ depends on the elements of I_t. That is, $h(Y_{t+n}, B)$ can be conditionally heteroscedastic or serially correlated, but there is no need to worry about the specific nature of these processes. Statistical inference can be conducted without explicitly characterizing the dependence of the conditional variances on the information set, although this will impact the relative efficiency of the estimators.

Second, these estimators can be thought of as being instrumental variables estimators, with Z_t as the vector of instruments which makes it possible to use ex-post observed instead of unobservable expected values of some of the variables entering the model and still obtain estimators with desirable properties. It is only required that the instruments be predetermined as of time period t; but they do not have to be "econometrically exogenous." Current and lagged values of Y can be used as instruments.

Third, the asymptotic covariance matrix for these estimators depends on the choice of W_T. It is possible to choose W_T such that the resulting estimators will have the smallest asymptotic covariance matrix among their class. Specifically, assume that h is differentiable, and that the vector of instruments Z_t is chosen such that the matrix

$$D_o = E \, [\, (\, dh \, (\, Y_{t+n} \, , \, B \,) \, / \, d \, b \,) \, .*. \, Z_t \,] \tag{2.17}$$

has full rank. Also assume that W_T converges almost surely to a limiting full rank matrix of constants W_o and let

$$S_o = \sum_{j=-n+1}^{n-1} E \, [\, F(\, Y_{t+n}, \, B, \, Z_t \,) \, F(\, Y_{t+n-j}, \, B, \, Z_{t-j} \,)' \,] \tag{2.18}$$

Assuming that S_o has full rank, Hansen (1982) shows that the smallest asymptotic covariance matrix for the estimator $\hat{b}$ is obtained by letting W_o be $W_o^* = (\, S_o \,)^{-1}$. The resulting asymptotic covariance matrix would be

(2.19) $$\text{cov}(\hat{b}) = (D_o' (S_o)^{-1} D_o)^{-1}.$$

Consistent estimators of these matrices can be obtained as follows:

(2.20) $$D_T = (1/T) \sum_{t=1}^{T} (dh (Y_{t+n} , \hat{b}) / db) .*. Z_t .$$

$$R_T (j) = (1/T) \sum_{t=1+j}^{T} (F (Y_{t+n} , Z_t , \hat{b})$$

$$F (Y_{t+n-j} , Z_{t-j} , \hat{b}) ').$$

$$W_T^* = \{ R_T (0) + \sum_{j=1}^{M} (R_T (j) + R_T (j) ') \}^{-1}$$

where M is the lag length considered "appropriate" for forming the instruments.

There is little guidance from econometric theory for choosing the appropriate lag lengths for forming the instruments. As Tauchen (1986) points out, results such as those of Hayashi and Sims (1983) are suggestive but not that helpful in practice, with a given fixed-sized data set, since they are based on an iterated-limit argument where the limit is first taken with respect to sample size and then with respect to the dimensionality of the instrument set.

In addition, as Whitney and West (1987) point out, if M is not equal to zero, the matrix W_T^* is not guaranteed to be positive semi-definite in any finite sample. This

characteristic interferes with asymptotic confidence interval formation and hypothesis testing. West (1985) proposes an estimator that is positive semi-definite for any finite sample:

$$(2.21) \qquad W_T^* = \{ R_T(0) + \sum_{j=1}^{M} G(j,M) (R_T(j) + R_T(j) ') \}^{-1}$$

where $G(j,M) = 1 - [j / (M-1)]$.

To implement (2.21), an initial consistent estimator of B is needed. Such an estimator can be obtained by using a "nonoptimal" choice of W_T to get the procedure started. Then, the initial consistent estimator can be used to compute the "optimal" W and the asymptotically "optimal" estimator of the parameter vector from a "second step" objective function. Using this procedure iteratively until convergence obviously will not improve the asymptotic properties of the final estimators, but could yield estimators with more desirable (but unknown) small sample properties.

Finally, Hansen (1982) suggests a straightforward way of testing the restrictions implied by the theoretical model. The estimation technique outlined above, sets l linear combinations of the r orthogonality conditions equal to zero as required by the first-order conditions for minimization of the criterion function (2.16):

$$(2.22) \qquad [(dG (b)'/ db) W_T] G_T (b) = 0$$

It follows that when $r > 1$, there are $r - 1$ remaining linearly independent orthogonality conditions that are not set to zero in estimation, but should be close to zero if the model restrictions are true. To test those overidentifying restrictions, a theorem in Hansen (1982) can be used. This theorem implies that T times the minimized value of (2.16) for an optimal choice of W_T is asymptotically distributed as a chi-square random variable with $r - 1$ degrees of freedom. It follows that the minimized value of the second-step objective function can be used to test how closely the first-order conditions generated by the theoretical model conform to reality. This can be interpreted as a statistical test to the validity of the theoretical model. There is little evidence regarding the performance of this test in small samples. Tauchen (1986) presents evidence from Monte Carlo simulations suggesting that the test performs reasonably well in moderate-sized samples. This assessment, however, refers to the validity of the size of the test. There is no information as to the performance of this test in terms of rejecting incorrect model specifications.

Estimation

Given the nature of this modeling effort, the generalized method of moments estimation procedure can be implemented based on two different information sets: the basic equation for the stochastic optimal control problem, or the optimal path equations for the endogenous variables.

The basic equation for the stochastic optimal control problem, in this case, does not require information regarding variables such as withdrawals from the firm and leverage position at every time period. The variables entering this equation are the expected rate of return to assets (which makes it a stochastic equation), the variance of the rate of return to assets, the equity value in the farm enterprise, and the cost of debt capital. This equation, however, does not provide as much information about the conditions to be fulfilled by a utility maximizing entrepreneur as the combined optimal path equations. This fact becomes clear when working through the solution procedure for the optimization problem.

The optimal path equations for consumption and the debt-to-asset ratio are both functions of the expected rate of return to assets (so that they are both stochastic and can therefore be used as a basis for generalized method of moments estimation), possibly all the other previously mentioned exogenous paths and of course, the unknown risk aversion coefficient b. If estimators for the risk aversion coefficients are to be obtained from those equations, information about consumption expenditures and leverage position for every time period is necessary. In this study, estimation will be based on the optimal path equations for consumption and the debt-to-asset ratio.

Maximum likelihood estimation of b could be based on the stochastic properties of the optimal path equations. It has

been assumed that the decision maker uses an expected value for the rate of return to assets in his quest for utility maximization. It has also been assumed that he perceives the rate of return to assets as being a normally distributed random variable.

It then follows that the observed (ex-post) rate of return to assets could be considered as measuring the expected rate of return to assets with a normally distributed error:

$$(2.23) \qquad \hat{R}_A\ (t) = R_A\ (t) + U_t \qquad\qquad U_t \sim N\ (\ 0\ ,\ \sigma_A\ )$$

This is equivalent to assuming that a very good estimate for the expected rate of return to assets is available to the decision maker. The realized value of $R_A(t)$ will then vary around this estimate according to a normally distributed error.

If equation (2.23) is substituted into the optimal path equations for consumption and leverage position, a full likelihood function could then conceptually be specified based on the stochastic properties of the error term in equation (2.23). Several other matters, however, still need to be considered. First of all, the expected rate of return to assets enters one of the optimal paths in a nonlinear fashion. Therefore, even under the simple assumption (2.23) regarding the stochastic properties or the expected rate of return to assets, the full likelihood function will exhibit a quite complex form. Second, besides errors in predicting the rate

of return to assets, there may be other types of disturbances that may cause the equations not to hold exactly even if the $R_A(t)$ could be predicted with absolute accuracy. If all these factors are taken into account, specifying a full information likelihood function becomes a cumbersome task and there is never a guarantee that all of the stochastic mechanisms associated with the optimal path equations have been properly specified.

Maximum likelihood estimation in this case requires substantially more information than the generalized method of moments. Since this information (specifically, the exact forms of the density functions of the random variables entering the system and the random errors associated with the optimal path equations) is necessary to estimate the unknown parameters, the estimators would be asymptotically more efficient if this additional information is correct. Amemiya (1974) notes, however, that maximum likelihood estimates may fail to be consistent if the distribution of the observable variables and the random errors is misspecified. The method of moments estimators are robust in the sense that the probability of obtaining inconsistent estimators is lower. The assumptions necessary to ensure the consistency of this type of estimators as noticed before, are very general. It is therefore my judgement that, given the nature and purpose of this study, GMM is the most appropriate strategy to follow.

Individual-Level vs Aggregate Behavior

One of the most challenging problems in economics relates to specifying theoretical models of aggregate behavior. Aggregate demand functions, for example, do not inherit most of the properties associated with the demand function of a single, utility-maximizing individual. As Varian (1984) points out, there is no aggregate version of Slutsky's equation or of the Strong Axiom of Revealed Preference. The aggregate demand function cannot, in general, be assumed to possess any interesting properties other than homogeneity and continuity. The neoclassical theory of the consumer places no restrictions on aggregate behavior in general.

As Deaton (1981) points out, there are no obvious grounds to argue that the theory, formulated for individual households (or firms), should be directly applicable to aggregate behavior. The transition from the microeconomics of consumer (or firm) behavior to the analysis of aggregate data is frequently referred to as the "aggregation problem." Aggregation is often viewed as a nuisance, a temporary obstacle lying on the way of straightforward application of the theory to the data.

The role of aggregation theory is to provide the necessary conditions under which it is possible to treat aggregate behavior as if it were the outcome of the decisions of a single utility-maximizing consumer (or profit-maximizing firm). These aggregation conditions often turn out to be very

stringent. Because of this, many economists "sweep the whole problem under the carpet" or dismiss it as of no importance.

Some economists (see Hicks, 1956, for example) have held the view that microeconomic theory has greater relevance for aggregate data. They argue on intuitive grounds that the variations in circumstances of individual economic units average out to negligible proportions in the aggregate, leaving only the systematic effects of variations in the exogenous variables.

This inherent conviction among economic theorists, possibly coupled with the lack of better alternatives, has lead to widespread use of microeconomic models and theories in situations where aggregate behavior is the issue. The linear expenditure system, for example, which is derived by algebraically imposing the theoretical restrictions of adding up, homogeneity, and symmetry to a linear formulation of demand has often been used in aggregate demand analysis. The Rotterdam model, in which the Slutsky equation is used to transform a double-logarithmic differential demand equation into a Hicksian demand equation, has also been extensively used in aggregate demand analysis. There has also been extensive research (see for example Barten, 1969, Deaton, 1974, Barten and Geyskens, 1975) on investigating whether the restrictions on demand implied by utility-maximizing behavior at the individual level hold in the aggregate. This reinforces the fact that there is an intrinsic belief among

economists that theories and relationships steming form the optimizing behavior of a single economic unit are useful and at least worthy of consideration as being consistent with aggregate behavior.

In this study, a theoretical model of the intertemporal optimizing behavior of an individual firm yields a set of relationships that are quite consistent with economic logic. Furthermore, while it is not possible to argue in strictly theoretical grounds that these relationships should hold in the aggregate level, it is not unreasonable to postulate these set of relationships as a probable model for aggregate reality.

Constant Absolute vs Constant Relative Risk Aversion

The nature of the objective function in intertemporal optimization problems such as the one proposed in this study plays a crucial role in determining the characteristics of their solution. In continuous formulations, individual decision makers are usually assumed to maximize the expected present value of an infinite stream of utility when given some exogenous discount rate. In general, utility functions implying constant absolute and constant relative risk aversion are the most commonly used types of objective functions within the context of risk analysis.

The Pratt-Arrow absolute risk aversion coefficient (Pratt, 1964) defined as $r(x) = -u''(x) / u'(x)$, has been used in many analyses which order alternative action choices

under conditions of uncertainty (Danok, McCarl, and White, 1980; King and Lybecker, 1983; Cochran, Robison, and Lodwick, 1985). It is characterized by being invariant with respect to linear transformations of the utility function but not to arbitrary rescalings of the outcome measure x. That is, the degree of risk aversion implied by a given value of r (x), will depend on the nature of the outcome variable. The reason for this dependence, as Raskin and Cochran (1986) point out, is that r (x) can be interpreted as the percent change in marginal utility per unit of outcome space. Therefore, r (x) has a unit of measurement associated with it so that the reciprocal of this measure is the unit with which the outcome space is being measured. As the scale in which the outcome variable is being measured is expanded, smaller values of the absolute risk aversion measure will be associated with the same level of risk aversion. For example, if the outcome variable is annual farm income (measured in dollars), a risk aversion measure between .001 and .0005 is usually considered as indicating strong risk aversion, while a measure between .0001 and .00001 is usually considered as indicating "almost" risk neutrality. On the other hand, if the outcome variable is 10-year after tax net present value (also measured in dollars), a risk aversion measure between .0001 and .000015 is usually considered as indicating strong risk aversion, while a measure between .00001 and .000001 is usually considered as

indicating "almost" risk neutrality (Raskin and Cochran, 1986).

Another issue is whether the specific functional form of a given utility function yields a Pratt-Arrow measure that is constant with respect to the value of the outcome variable x. If this is the case, the utility function is said to imply constant absolute risk aversion. In terms of attitudes towards risk, constant absolute risk aversion will imply (in the discrete case) that, even when all other relevant factors remain the same (such as expected value and variance of the contract), a contract that is acceptable for certain values of the outcome variable may not be acceptable for some other values of the outcome variable, and vice versa.

The relative risk aversion coefficient is defined as $r^* (x) = r (x) * x$, where x is an element of the outcome space. While r measures the percent change in marginal utility per unit change of the outcome space, r^* measures the same marginal utility change per percent change of the outcome space. The relative risk aversion coefficient is therefore the elasticity of the marginal utility function and as such is unitless. Relative risk aversion measures of around 6 are usually considered as indicating strong risk aversion, while measures closer to zero indicate a more risk neutral kind of behavior (Raskin and Cochran, 1986).

A utility function that yields a measure of relative risk aversion r^*, that is constant relative to the value of the

outcome variable, is said to imply constant relative risk aversion. In terms of attitudes towards risk, constant relative risk aversion will imply (in the discrete case) that the decision of whether a specific contract will be taken or not is completely independent of the value of the outcome variable at the time the contract is being considered.

CHAPTER 3
RESULTS

The Solutions of the Stochastic Optimal Control Problems

Constant Relative Risk Aversion

A detailed account of the procedures and mathematical techniques required to solve the stochastic optimal control problems is given the appendix. The optimal control equations (endogenous paths) for the leverage position and consumption level under the assumption of constant relative risk aversion are[1]

$$\delta^*(t) = 1 - (1-b)\sigma_A^2(t)/(\hat{R}_A(t)-K(t)) \quad , \text{ and} \tag{3.1}$$

$$C^*(t) = E^*(t)(Ab)^{1/b-1} \quad , \tag{3.2}$$

where

$$Ab = \frac{\{r(t)-K(t)b-(K(t)-\hat{R}_A(t))^2 b/2\sigma_A^2(t)(1-b)\}^{(1-b)}}{(1-b)} \tag{3.3}$$

Sufficient conditions for (3.1) and (3.2) to be the optimal paths associated with this stochastic optimal control

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[1]Throughout the results section, b will be referred to as the relative risk aversion measure. The Raskin and Cochran definition of such measure, however, is b_r=1-b. Therefore, a value of b=-5 implies strong risk aversion, while values approaching 1 (from below) imply more risk neutral types of behavior.

problem are that the rate of return to assets exceeds the cost of debt capital and that (1-b) is positive (implying a risk averse decision maker). If either of these conditions does not hold, (3.1) and (3.2) cannot be considered to be the optimal paths for the debt-to-asset ratio and consumption.

Equation (3.1) essentially replicates in dynamic terms the static results obtained by Collins (1985) using a DuPont expansion. In Collins' model, optimal leverage increases with an increase in the expected total rate of return to assets, declines with an increase in the variance of the rate of return to assets, and declines with an increase in the cost of debt capital.

These same results can be derived within the dynamic framework by taking the total differential of equation (3.1),

$$(3.4) \qquad d\delta^*(t) = \frac{(1-b)\sigma_A^2(t)}{(\hat{R}_A(t)-K(t))} d\hat{R}_A(t) - \frac{(1-b)^2}{(\hat{R}_A(t)-K(t))} d\sigma_A^2(t) - \frac{(1-b)\sigma_A^2(t)}{(\hat{R}_A(t)-K(t))} dK(t)$$

Setting $d\sigma_A^2(t) = dK(t) = 0$,

$$(3.5) \qquad d\delta^*(t)/dt = \frac{(1-b)\sigma_A^2(t)}{(\hat{R}_A(t)-K(t))} d\hat{R}_A(t)/dt$$

An increase in the expected rate of return to assets will result in an increase in the optimal leverage position

over time, ceteris paribus. The magnitude of the relative change is directly proportional to the variance of the rate of return on assets and the degree of risk aversion of the decision maker, and it is inversely related to the difference between the expected total rate of return to assets and the cost of debt capital.

Similarly, letting $d\hat{R}_A(t) = dK(t) = 0$,

$$(3.6) \qquad d\delta^*(t)/dt = \frac{(1-b)^2}{(\hat{R}_A(t)-K(t))} \; d\sigma_A^2(t)/dt$$

An increase in the variance of the total rate of return to assets will result in a lower optimal debt-to-asset ratio over time. The magnitude of the percentage change does not depend on the variance of the total rate of return to assets in this case, rather it is directly proportional to the degree of risk aversion and inversely related to the difference between the expected rate of return on assets and the cost of debt.

Setting $d\hat{R}_A(t) = d\sigma_A^2(t) = 0$

$$(3.7) \qquad d\delta^*(t)/dt = \frac{(1-b)\sigma_A^2(t)}{(\hat{R}_A(t)-K(t))} \; dK(t)/dt$$

A decrease in the cost of debt capital will lead to an increase in the optimal level of debt through time. Notice that the magnitude of the relative change is exactly the same as that associated with $d\hat{R}_A^2(t)$. That is, if the expected rate

of return to assets and the cost of debt capital both change by the same relatively small amount, the optimal debt-to-assets ratio will remain the same.

Finally, since a smaller (more negative) relative risk aversion coefficient implies greater risk aversion, the greater the degree of risk aversion exhibited by the entrepreneur the lower optimal leverage. Thus, the basic results of Collins (1985a) are supported within the framework of a stochastic optimal control formulation.

An interesting feature of the optimal path for the debt-to-asset ratio under constant relative risk aversion is that it is not a function of farm-equity value. That is, farm size does not affect the optimal leverage position of the operation. A long-term trend in the U.S. agricultural sector has been one of a decreasing number of farms (Agricultural Statistics, 1990). In addition, the farms remaining in business have become larger. For the typical farm, equation (3.1) implies a stationary leverage position over time if the expected rate of return to assets, the variance of the rate of return to assets and the cost of debt capital are stationary exogenous paths. Since the leverage position for the typical farm operation should be stationary even as farm-level equity value increases over time. Equation (3.1), therefore, could be expected to reasonably describe the aggregate behavior of the agricultural sector, if constant relative risk aversion holds at the farm level.

Another issue related to the optimal leverage position given by equation (3.1) is that of financial risk. The debt-to-asset ratio is considered as an important indicator of financial risk by credit institutions. It has also been regarded as a parameter in the probability distribution of bankruptcy by some finance theorists (Karp and Collins). Since most farm income support programs involve provisions that affect both the expected rate of return to assets and the variance of the rate of return to assets, it would be interesting to compare the effects of changes of such exogenous paths on the leverage position of the firm as well as on the farmer's welfare as reflected by his yearly consumption path. Because of the debt related problems that have afflicted the agricultural sector during the decade of the 80's, it may be of special interest to evaluate the hypothetical consequences of policies that aim to maintain the leverage position at a constant level over time while motivating the entrepreneur to withdraw sufficient income from the farm operation to achieve a socially acceptable standard of living.

By rearranging the total differential of $\delta*$ (equation (3.4)), setting $d\delta*(t) = dK(t) = 0$, solving for $d\hat{R}_A(t)/d\sigma_A^2$, and multiplying by $\sigma_A^2/\hat{R}_A$, it can be shown how it is possible to keep the optimal debt-to-asset ratio constant while the expected rate of return to assets and the variance of the rate of return to assets are changed over time

$$(3.8) \qquad d\hat{R}_A(t)/d\sigma_A^2(t) = (\hat{R}_A(t)-k(t))/\sigma_A^2$$

Equation (3.8) could be used, in conjunction with equation (3.2) (or comparative statics results regarding equation (3.2)), to evaluate the impact of such policies on variables such as consumption rate, equity level and asset-value holdings. Theoretically, if the expected rate of return to assets and the variance of the rate of return to assets are changed according to equation (3.8), the debt-to-asset ratio should remain constant over time.

Given current values of the exogenous and endogenous variables, equation (3.8) could also be used in conjunction with equation (3.2) and equation (2.8) (which specifies the evolution of equity value over time as a function of the control variables and exogenous paths) to attempt an inter-temporal simulation of the endogenous control and state variables. Results from intertemporal simulation can also be used to evaluate the effect income support policies (implying changes in the expected rate of return to assets and the variance of the rate of return to assets that are consistent with equation (3.8)) on farm equity evolution as well as on the farmer's welfare as reflected by his yearly consumption path over time. Theoretically, such policies would be neutral with respect to the leverage ratio.

Equation (3.2) states that the optimal consumption rate is directly proportional to the optimal level of equity at time period t; with a proportionality constant that depends on all the exogenous paths and the level of risk aversion of the

decision maker. It is important to note that, given an initial level of equity and constant values of the expected rate of return to assets, the variance of the rate of return to assets, the cost of debt capital, and the discount rate, the optimal expected levels of consumption and equity will increase over time. Also, the actual future values taken by these variables are random since both processes depend on equation (2.9) which itself is random.

Even though the optimal consumption rate depends on the optimal equity level through the proportionality constant which is a function of the exogenous paths, the main effect of changes in any of the exogenous paths on the optimal consumption rate and equity level is always through equation (2.9) which stochastically governs the optimal evolution of equity over time. Changes in the proportionality constant (equation (3.3)) are dominated by this main effect in the long run. It thus follows that the expected total differential of equation (2.9) is the key to analyzing the effects of changes in the exogenous paths on the expected optimal level of consumption.

The differential of the expected change in equity over time can be defined as

$$(3.9) \qquad dExp(dE^*(t)) = 2E^*(t) \left(\frac{1}{1-\delta^*(t)}\right) d\hat{R}_A(t) - E^*(t) \left(\frac{1}{1-\delta^*(t)}\right)(1-b)^2 d\sigma_A^2(t) - 2E^*(t)\left(\frac{1}{1-\delta^*(t)}\right) dk(t).$$

Examination of equation (3.9) reveals that an increase in the expected rate of returns to assets will accelerate the optimal evolution of equity over time. This acceleration will result in higher expected optimal paths for consumption and equity in the long run. A change in the cost of debt capital, K(t), will have a long-run effect similar in magnitude but opposite in sign to that of a change in the expected total rate of returns to assets. Finally, an increase in the variance of the total rate of returns to assets will slow the optimal evolution of equity over time, resulting in lower expected optimal paths for consumption and equity.

The effect of a change in the expected rate of return to assets, the variance of the rate of return to assets or the cost of debt capital on the optimal consumption rate is a result of two separate adjustments. An adjustment in the optimal debt-to-asset ratio and an adjustment in the optimal consumption-investment rate decision. For example, a decrease in the variance of the total rate of returns to assets will result in a higher optimal leverage position. Even though the average total rate of return to assets to be obtained from the farm operation over time has not changed, the farmer will now control more productive assets, which will result in a higher total return from these assets. The long-run adjustment in the consumption-investment rate decision will be such that at least part of these additional returns will eventually be consumed. An increase in the expected total rate of returns

to assets will result not only in a higher optimal leverage position but, if the farmers' expectations are correct, in a higher average total rate of return to assets to be obtained from the farm operation. The long-run adjustment in the consumption-investment rate decision, however, will always be such that some of the additional returns will eventually be consumed.

Even more important than the optimal consumption path over time, may be the optimal path of the average propensity to consume. The average propensity to consume disposable income is

$$(3.10) \qquad APC(t) = C(t)/(\{[\hat{R}_A-(k*\delta)]*(1/(1-\delta)\}E(t))$$

$$= (Ab)^{1/(b-1)} \{[\hat{R}_A-(k*\delta)]*(1/(1-\delta)\}^{-1}$$

where $\{[\hat{R}_A-(k*\delta)]*(1/(1-\delta)\}$ is the expected rate of return to equity, as defined in equation (2.7). Since the optimal debt-to-asset ratio does not depend on equity value under constant relative risk aversion, increasing farm-level equity values should, theoretically, have no effect on the optimal path of the average propensity to consume. That is, as the typical farmer holds more equity (becomes wealthier), he will not tend to consume a lower (or higher) proportion of his disposable income. Therefore, the long-run trend of an agricultural sector where the typical farm is becoming larger should have no effect on the aggregate average propensity to consume.

Constant Absolute Risk Aversion

The optimal control equations (endogenous paths) for the leverage position and consumption level, under the assumption of constant absolute risk aversion, are (see appendix)

$$\delta^*(t) = 1 - (\lambda K(t)\sigma_A^2(t)E(t))/(\hat{R}_A(t)-K(t)) \tag{3.11}$$

$$C^*(t) = K(t)E(t) + \frac{[r(t)-K(t)+(\hat{R}_A(t)-K(t))^2/2\sigma_A^2(t)]}{\lambda K(t)} \tag{3.12}$$

The optimal path equation for the debt-to-asset ratio is a decreasing function of λ. Since λ is the Pratt-Arrow measure of risk aversion, equation (3.11) implies that as long as the expected rate of return to assets exceeds the cost of debt capital less risk-averse individuals will take more debt relative to their total asset holdings. Notice that under this circumstance, as λ approaches zero (the individual becomes risk neutral), the optimal debt-to-asset ratio approaches a value of one. That is, the individual would prefer to finance most of the operation with debt. Equations (3.11) and (3.12) cannot be considered the optimal paths for the debt-to-asset ratio and consumption either for negative values of λ (risk loving behavior), or when the cost of debt capital exceeds the expected rate of return to assets. Under these circumstances, sufficient conditions for (3.11) and (3.12) to be a solution of the stochastic optimal control problem are violated.

The optimal debt-to-asset ratio is also a decreasing function of the cost of debt capital K(t), since

$$(3.13) \quad d\delta^{*}(t)/dK(t) = -[\{K(t)/(\hat{R}_A(t)-K(t))\}+1]$$

$$[(\lambda\sigma_A^2(t)E(t))/(\hat{R}_A(t)-K(t))]$$

is less than zero, and an increasing function of the expected rate of return to assets, since

$$(3.14) \quad d\delta^{*}(t)/d\hat{R}_A(t) = (\lambda K(t)\sigma_A^2(t)E(t))/(\hat{R}_A(t)-K(t))^2]$$

is greater than zero.

Notice, however, that the effect of an infinitesimal change in the cost of debt capital on the optimal debt-to-asset ratio dominates the effect of an infinitesimal change in the expected rate of return to assets. That is, if the expected rate of return to assets and the cost of debt capital both increase (decrease) by the same relatively small amount, the optimal debt-to-asset ratio will decrease (increase). This asymmetric response does not occur under constant relative risk aversion.

The optimal debt-to-asset ratio is a decreasing function of the variance of the rate of return to assets. As in the case of constant relative risk aversion, it may be of interest to compare the effects of changes of exogenous paths such as the expected rate of return to assets and the variance of the rate of return to assets on the leverage position of the firm as well as on the farmer's welfare as reflected by his yearly

consumption path. It may be of special interest to evaluate the hypothetical consequences of a policy that aims to maintain the leverage position at a constant level over time while motivating the entrepreneur to withdraw sufficient income from the farm operation to achieve a socially acceptable standard of living. Taking the total differential of equation (3.11), setting $d\delta^*(t) = dK(t) = dE(t) = 0$, solving for $d\hat{R}_A(t)/d\sigma_A^2(t)$ and multiplying by $\sigma_A^2(t)/\hat{R}_A(t)$ yields

$$(3.15) \qquad d\hat{R}_A(t)/d\sigma_A^2(t) = (\hat{R}_A(t)-k(t))/\sigma_A^2$$

Equation (3.15) gives the change in the expected rate of return to assets that has to accompany a small change in the variance of the rate of return to assets so that the leverage position of the firm will remain constant. It is interesting to notice that equation (3.15) is identical to equation (3.8), its counterpart under constant relative risk aversion. As in the case of constant relative risk aversion, equation (3.15) can be used in conjunction with equation (3.12) and equation (2.8) (which specifies the evolution of equity value over time as a function of the control variables and exogenous paths), to attempt an intertemporal simulation of the endogenous control and state variables. Results from intertemporal simulation can be used to evaluate the effect income support policies (imposing changes in the expected rate of return to assets and the variance of the rate of return to assets that are consistent with equation (3.5)) on farm equity evolution

as well as on the farmer's welfare as reflected by his yearly consumption path over time. Theoretically, such policies would be neutral with respect to the leverage ratio.

Another interesting and unique feature of the optimal path equation for the debt-to-assets ratio under constant absolute risk aversion is that it is a function of equity value. Farms with larger equity value should, theoretically, take less debt relative to their total asset value, if everything else remains constant. A long-term trend in the U.S. agricultural sector has been one of a decreasing number of farms (Agricultural Statistics, 1990). In addition, the farms remaining in business have become larger. Therefore, for the typical farm, equation (3.11) implies an increasingly more conservative leverage position over time, if the expected rate of return to assets, the variance of the rate of return to assets and the cost of debt capital are stationary exogenous paths. Since the typical leverage position should be more conservative as farm-level equity value increases over time, the aggregate leverage position should also be a decreasing function of aggregate equity value, after it has been corrected for the randomly fluctuating exogenous paths. Equation (3.11), therefore, could be expected to reasonably describe the aggregate behavior of the agricultural sector, if constant absolute risk aversion holds at the farm level.

Evaluating the effect of changes in the exogenous paths on present and future consumption at the individual level is

not such a simple task. Equation (3.12) indicates that the optimal consumption rate is directly proportional to the optimal level of equity at time period t. As in the case of constant relative risk aversion, given an initial level of equity and constant values of the expected rate of return to assets, the variance of the rate of return to assets, the cost of debt capital and the discount rate, the optimal expected levels of consumption and equity will increase over time.

Under constant absolute risk aversion, the optimal consumption rate is linearly related to the equity level, with the cost of debt capital as slope coefficient and a function of all the exogenous paths and the risk aversion coefficient serving as an intercept. However, as in the case of constant relative risk aversion, the main effect of changes in any of the exogenous paths on the optimal consumption rate and equity level is always through equation (2.9) which stochastically governs the optimal evolution of equity over time (Ramirez et al., 1990). Changes in the slope or the intercept of (3.2) are dominated by this main effect in the long run.

Taking the total differential of equation (2.9) in this case, yields a very complex function of equity value and the exogenous paths than can not be easily simplified. The resulting differential is not useful for making inferences regarding the effects of changing any of the exogenous paths on the optimal equity evolution and consumption rate over time. The increased complexity is due to the functional form

of the optimal path for consumption under constant absolute risk aversion. Even though the kind of differential analysis conducted for the case of constant relative risk aversion is not possible, intertemporal simulation reveals that an increase in the expected total rate of returns to assets will accelerate the optimal evolution of equity over time. This acceleration will result in higher expected optimal paths for consumption and equity in the long run. A change in the cost of debt capital, K(t), will have a long-run effect of greater magnitude and opposite in sign to that of a change in the expected total rate of returns to assets. Finally, an increase in the variance of the total rate of returns to assets will slow the optimal evolution of equity over time, resulting in lower expected optimal paths for consumption and equity. These results are similar to those obtained under constant relative risk aversion.

The average propensity to consume under constant absolute risk aversion is

$$(3.16) \qquad C_{AV}(t) = \frac{K(t)+[r(t)K(t)+(\hat{R}_A(t)K(t))^2/2\sigma_A^2(t)]/\lambda K(t)E(t)}{\hat{R}_e(t)}$$

where $\hat{R}_e(t) = \{[\hat{R}_A(t)-(k(t)\delta(t))](1/(1-\delta(t))\}$ is the expected rate of return to equity. Notice that in this case the rate of return to equity is a function of equity value, since the optimal debt-to-asset ratio depends on the actual equity level. Specifically

$$(3.17) \qquad d\hat{R}_e(t)/dE(t) = -[K(t)/(1-\delta(t))]d\delta(t)/dE(t) + [\{\hat{R}_A(t)-k(t)\delta(t)\}/(1-\delta(t))^2]d\delta(t)/dE(t)$$

Even though $d\delta(t)/dE(t)$ is known to be negative, the sign of $d\hat{R}_e(t)/dE(t)$ is ambiguous. The direct effect of increasing equity values on the average propensity to consume, however, is negative. That is, if the rate of return to equity is assumed stationary, as the typical farmer holds more equity (becomes wealthier), he will tend to consume a lower proportion of his disposable income. Therefore, the long-run trend of an agricultural sector where the typical farm is becoming larger would have the effect of decreasing the aggregate average propensity to consume.

Estimation and Hypothesis Testing

The solution of the stochastic optimal control problem under the assumptions of constant relative and constant absolute risk aversion, yields two sets of optimal paths for the debt-to-asset ratio and consumption. The optimal paths for the leverage position are functions of the expected rate of return to assets, the cost of debt capital, the variance of the rate of return to assets, the corresponding risk aversion parameters, and equity value in the case of constant absolute risk aversion. The optimal paths for consumption are in both cases functions of all of the exogenous paths including the discount rate, the corresponding risk aversion parameters, and equity value.

The farmer's expectation of the rate of return to assets is obviously unobservable. The value of this random variable, however, may be inferred after all the withdrawal-reinvestment and financing decisions for the time period under consideration have been made. If the generalized method of moments estimator is used, corresponding realizations can be used instead of the farmer's expectations, without compromising the consistency and asymptotic normality of the estimator. Using the realized instead of the expected rate of return to assets is no different, from a statistical perspective, than using a proxy variable for the expected rate of return to assets, if the expected value of the proxy variable were indeed the expected rate of return to assets.

Therefore, if the inflation rate can be assumed to approximate the farmer's discount rate with an expected error of zero, it can be used instead of the discount rate without compromising the consistency and asymptotic normality of the generalized method of moments estimator. Since nominal (non-deflated) data are being used, the inflation rate can be assumed to be a reasonable proxy variable for the discount rate at the aggregate level.

A related problem arises regarding the variance of the rate of return to assets. The farmers' perception of the variance of the rate of return to assets can be assumed to follow the following process

$$(3.18) \qquad \sigma_A^2(t) = [\sum_{j=0}^{K} (\hat{R}_A(t-j)-\hat{R}_{At})^2 / K] + e_t, \quad t=1+K, \ldots, T.$$

where $\hat{R}_{At} = \sum_{j=0}^{K} (R_A(t-j)) / K$, $t=1+K, \ldots, T$, and the expected value of e_t is zero.

That is, the farmer's perception of the variance of the rate of returns to assets can be approximated by the average of the square of the differences between the actual rates of return to assets and their average for the previous K time periods. This approach assumes that the farmer takes into account the variability of the rate of return to assets during the recent past. If (3.18) can be assumed to hold, $\hat{\sigma}_A^2(t) = [\sum_{j=0}^{K} (\hat{R}_A(t-j)-\hat{R}_{At})^2 / K]$ can be used as a proxy variable for the farmer's perception of the variance of the rate of return to assets.

Another issue that needs consideration before attempting to estimate the parameters entering the optimal path equations is the choice of the variables to be used as instruments in the construction of sample versions of the population orthogonality conditions implied by such optimal paths. Hansen and Sargent and Hayashi and Sims provide extensive discussions of optimal instrument selection in linear environments. The optimal path equations, however, are highly non-linear functions of the exogenous paths, state variable and risk aversion coefficients. No optimal instrument selection criteria is available outside of linear environments.

Therefore, it was decided a priori to use the debt-to-asset ratio, cost of debt capital, consumption rate and equity level as instrumental variables in all estimation efforts.

It is also necessary to make a decision regarding the lag length for forming the instruments. There is little guidance from econometric theory for choosing the appropriate lag lengths. Tauchen (1986) points out that previous findings (Hayashi and Sims, 1983) are suggestive but not that helpful for application with a data set of fixed size, because they are based on an iterated-limit argument where the limit is first taken with respect to sample size and then with respect to the dimensionality of the instrument set. Therefore, it is decided (also a priori) to use a moderate lag length of four in all estimation efforts.

Once all the issues regarding expectations, unobservable variables, instrument choice, and lag length have been dealt with estimation of the parameters (risk aversion coefficients) entering the optimal path equations can be attempted using aggregate data. Specifically, annual post-war data for the agricultural sector, excluding farm households will be used.

Under constant relative risk aversion, limited information (single equation) estimation is first attempted. Using the optimal path equation for the leverage position an initial (first-step) estimate of b, can be obtained as follows:

$$(3.19)\qquad V_1 = \sum_{t=1}^{T} [\ I(\delta_t)\ \delta_t\ ] \ /\ T$$

$$V_2 = \sum_{t=1}^{T} [\ I(\delta_t)\ K_t\ ] \ /\ T$$

$$V_3 = \sum_{t=1}^{T} [\ I(\delta_t)\ C_t\ ] \ /\ T$$

$$V_4 = \sum_{t=1}^{T} [\ I(\delta_t)\ E_t\ ] \ /\ T$$

where $I(\delta_t) = [\delta_t - 1 + \{(1-b)\sigma_{At}^2/(\hat{R}_{At}-K_t)\}]$, the optimal path for the leverage position, in implicit form.

Then, create the 1 by 4 vector-function of b, V = [V1 V2 V3 V4], and minimize the quadratic form given by Q = V W V', with respect to b. The 4 by 4 matrix W, is arbitrarily chosen to be a matrix of ones. Since Q is a function of only one parameter in this case, it can be easily minimized using a "grid search" procedure. Constant relative risk aversion measures usually range between -10 and 1. Even though the value of b that minimizes the given quadratic form is a consistent (but not an efficient) estimator for the true underlying measure of risk aversion, so that it is expected to fall within that range, the grid search is conducted for values of b from -50 to 50. The minimizing value of b, $\hat{b}$ is equal to -4.2420. The minimum value of the quadratic form is zero.

Given this initial consistent estimate of b, the optimal weighting matrix W^* can be computed as follows:

$$(3.20) \qquad l_1 = I(\delta) * \delta \qquad l_2 = I(\delta) * K$$

$$l_3 = I(\delta) * C \qquad l_4 = I(\delta) * E$$

where $I(\delta) = \delta-1+\{(1-\hat{b})\sigma_A^2/(\hat{R}_A-K)\}$, is the implicit form for the optimal path of the leverage position, in vector notation. Thus, δ, σ_A^2, $\hat{R}_A$, K, C, and E are T by 1 vectors. Furthermore, / and * are element by element operators so that l_1, l_2, l_3, l_4, are also T by 1 vectors. Define the T by 4 matrix $l = [l_1\ l_2\ l_3\ l_4]$, and

l11 = l[1:T-1,.]   l12 = l[2:T,.]

l21 = l[1:T-2,.]   l22 = l[3:T,.]

l31 = l[1:T-3,.]   l32 = l[4:T,.]

l41 = l[1:T-4,.]   l42 = l[5:T,.]

where l[i:j,.] is a matrix that includes from the i^{th} to the j^{th} row of l, and all of its columns. Then define M_0=(l'l)/T, M_1 = (l11'l12)/T, M_2 = (l21'l22)/T, M_3 = (l31'l32)/T, and M_4 = (l41'l42)/T.

Finally, let $W_i = [1-i/(m+1)]\ (M_i + M_i')$, (i=1,..4), where m is the lag length being used, which in this case equals 4. The optimal weighting matrix can now be computed as $W^* = (M_0+W_1+W_2+W_3+W_4)^{-1}$

In order to obtain an asymptotically efficient estimator of b, $Q^* = V\ W^*\ V'$ has to be minimized with respect to b.

Notice that V = [V1 V2 V3 V4], and V_i (i=1, . . ., 4) is exactly as given in (3.19). That is, the second step is just a repetition of the first step, using W^* instead of W as the weighting matrix.

A grid search for values of b ranging from -40 to 40 yields a estimate of $\tilde{b}$ of -2.0830, with the quadratic form reaching a minimum value of .1293. As pointed out in chapter 3, a theorem by Hansen (1982) implies that T times the minimized value of Q^* is, under the null hypothesis that the population orthogonality conditions implied by the optimal paths are indeed correct, asymptotically distributed as a chi-square random variable with r-l degrees of freedom (r being the number of orthogonality conditions used for estimation and l the number of parameters to be estimated). Therefore, when the test statistic T x Min (Q^*) = 4.6548, is compared with a chi-square random variable with 3 degrees of freedom, it is concluded that the relationships implied by the optimal path for the debt-to-asset ratio under constant relative risk aversion can not be shown to contradict the data generating mechanism, using Hansen's asymptotic test.

Furthermore, the estimate of the asymptotic standard error of $\tilde{b}$ can be computed (see Chapter 2), letting

(3.21) $$D_t = d[I_{(\delta t)}]/db = \sigma_{At} / (\hat{R}_{At} - K_t)$$

where again, $I(_{\delta t})$ refers to the implicit form of the optimal path equation for the leverage position at time period t. And

$$D_1=\sum_{t=1}^{T}(D_t\delta_t)/T \qquad D_2=\sum_{t=1}^{T}(D_tK_t)/T$$

$$D_3=\sum_{t=1}^{T}(D_tC_t)/T \qquad D_4=\sum_{t=1}^{T}(D_tE_t)/T$$

$$D = [D_1\ D_2\ D_3\ D_4]$$

Then, the asymptotic estimate of the standard error can then be computed as (since only one parameter is being estimated) $SE = (D\ W^*\ D')^{-1/2} = 13.0289$. In addition, since $\tilde{b}$ is strongly consistent and asymptotically normal, a 96% asymptotic confidence interval for b can be constructed by adding and subtracting twice the estimate of its asymptotic standard error to $\tilde{b}$. Therefore, the true underlying measure of risk aversion b, can be expected (asymptotically) to be between -28.1408 and 23.9748 with a 96% level of statistical confidence. Being able to place the true underlying measure of risk aversion between those bounds has, however, no empirical relevance. Risk loving behavior, for example, cannot be ruled out, and a relative risk aversion coefficient of -28 implies behavior beyond the boundaries of extreme risk aversion.

Estimation can also be attempted using the optimal path equation for consumption, under constant relative risk aversion, following (3.19), where

$$(3.22)\qquad I(C_t) = (C_t/E_t) - \{[(r_t - bK_t)/(1-b)] - [b(\hat{R}_{At} - K_t)^2/(2\sigma^2_{At}(1-b)^2)]\}$$

has to be used instead of $I(\delta_t)$.

A grid search for values of b between -50 and 50 was conducted, seeking to minimize the corresponding quadratic form and obtain a first-step estimate of b. Surprisingly, the quadratic form achieves a minimum value of zero for two different values of b, $\hat{b}_1$ = -.0004143 and $\hat{b}_2$ = -47.7738. Therefore, following (3.20), two optimal weighing matrices, W^*_1 and W^*_2, can be computed. There is no reference in the literature on how to deal with cases where multiple optimal weighting matrices are encountered. The most logical way to proceed is to compute the second-step estimates associated with both optimal weighting matrices and regard the one corresponding to the second-step quadratic form achieving the lower value as the most efficient estimate. The two second-step estimates are $\tilde{b}_1$ = -.000785 and $\tilde{b}_2$ = -37.1160, with their associated second-step quadratic forms achieving minimum values of .1706 and .1117, respectively. Therefore, $\tilde{b}$ = -37.1160 is to be considered the most efficient estimate of b. Also, when T times the minimum value of the quadratic form associated with the most efficient estimate (which equals 3.5744) is compared with a chi-square random variable with 3 degrees of freedom, it is concluded that the relationships implied by the optimal path for consumption under constant

relative risk aversion can not be shown to contradict the data generating mechanism, using Hansen's asymptotic test.

In addition, following (3.21), substituting in

$$(3.23)\qquad d[I(C_t)]/db = [(1/(1-b))^2-(2b/(1-b))^3][(\hat{R}_{At}-K_t)^2/2\sigma^2_{At}]$$
$$+[K_t/(1-b)]-[(r_t-bK_t)/(1-b)^2]$$

for $d[I(\delta_t)]/db$, the estimate of the asymptotic standard error of $\tilde{b}$ is found to be 15.3184. Although the bounds of a 96% asymptotic confidence interval for the true underlying measure of risk aversion, given by $-6.4792 > b > -67.7528$ allow ruling out risk loving behavior, they do not allow to place b within an empirically meaningful subset of its parameter space. Furthermore, a set of Monte Carlo studies conducted by Tauchen (1986), suggest that the generalized method of moments estimator of the curvature parameter of the constant relative risk aversion utility function can be biased in certain not unrealistic circumstances with the magnitude of the bias being as large as the asymptotic standard error. Such finding makes the single equation estimates of b seem even more irrelevant, from an empirical standpoint.

Full information (multiple equation) generalized method of moments estimation under the assumption of constant relative risk aversion can be attempted using both the optimal path for the leverage position and consumption. Define

$$(3.24) \qquad V_1 = \sum_{t=1}^{T} [\ I(\delta_t)\ \delta_t\ ]\ /\ T$$

$$V_2 = \sum_{t=1}^{T} [\ I(\delta_t)\ K_t\ ]\ /\ T$$

$$V_3 = \sum_{t=1}^{T} [\ I(\delta_t)\ C_t\ ]\ /\ T$$

$$V_4 = \sum_{t=1}^{T} [\ I(\delta_t)\ E_t\ ]\ /\ T$$

where $I(\delta_t) = [\delta_t - 1 + \{(1-b)\sigma_{At}^2/(\hat{R}_{At}-K_t)\}]$, the optimal path for the leverage position in implicit form, and

$$V_5 = \sum_{t=1}^{T} [\ I(C_t)\ \delta_t\ ]\ /\ T$$

$$V_6 = \sum_{t=1}^{T} [\ I(C_t)\ K_t\ ]\ /\ T$$

$$V_7 = \sum_{t=1}^{T} [\ I(C_t)\ C_t\ ]\ /\ T$$

$$V_8 = \sum_{t=1}^{T} [\ I(C_t)\ E_t\ ]\ /\ T$$

$I(C_t) = (C_t/E_t) - \{[(r_t - bK_t)/(1-b)] - [b(\hat{R}_{At}-K_t)^2/(2\sigma_{At}^2(1-b)^2)]\}$, the optimal path for consumption, in implicit form.

Then, create the 1 by 8 vector-function of b, $V_f = [V_1\ V_2\ V_3\ V_4\ V_5\ V_6\ V_7\ V_8]$, and minimize the quadratic form given by $Q_f = V\ W\ V'$, with respect to b. The 8 by 8 matrix W, as in the single equation case, is arbitrarily chosen to be a matrix of

ones. Since Q_f is a function of only one parameter in the multiple equation case as well, it can also be minimized using a grid search procedure. A grid search is conducted for values of b ranging from -50 to 50, and the quadratic form Q_f achieves a single minimum of zero for a value of b, $\hat{b}$ = -1.4308. The optimal weighing matrix can now be computed as follows:

(3.25) $$l_1 = I(\delta) * \delta \qquad l_2 = I(\delta) * K$$

$$l_3 = I(\delta) * C \qquad l_4 = I(\delta) * E$$

$$l_5 = I(C) * \delta \qquad l_6 = I(C) * K$$

$$l_7 = I(C) * C \qquad l_8 = I(C) * E$$

where $I(\delta) = \delta - 1 + \{(1-\hat{b})\sigma_A^2/(\hat{R}_A - K)\}$, is the implicit form for the optimal path of the leverage position in vector notation, and $I(C) = (C/E) - \{[(r-bK)/(1-b)] - [b(\hat{R}_A - K)^2/(2\sigma_A^2(1-b)^2)]\}$ is the implicit form for the optimal consumption path, also in vector notation.

Thus, δ, r, σ_A^2, $\hat{R}_A$, K, C, and E are T by 1 vectors. Furthermore, / and * are element by element operators so that l_i, (i=1, . . ., 8), are also T by 1 vectors. Define the T by 8 matrix $l = [l_1, l_2, l_3, l_4, l_5, l_6, l_7, l_8]$, and

```
l11 = l[1:T-1,.]     l12 = l[2:T,.]
l21 = l[1:T-2,.]     l22 = l[3:T,.]
l31 = l[1:T-3,.]     l32 = l[4:T,.]
l41 = l[1:T-4,.]     l42 = l[5:T,.]
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where l[i:j,.] is a matrix that includes from the i^{th} to the j^{th} row of l, and all of its columns. Then define $M_0 = (l'l)/T$, $M_1 = (l11'l12)/T$, $M_2 = (l21'l22)/T$, $M_3 = (l31'l32)/T$, and $M_4 = (l41'l42)/T$.

Finally, let $W_i = [1-i/(m+1)]\ (M_i + M_i')$, (i=1, . . ., 4), where m is the lag length being used, which in this case equals 4. The optimal weighting matrix can now be computed as $W* = (M_0+W_1+W_2+W_3+W_4)^{-1}$

In order to obtain an asymptotically efficient estimator of b, $Q^* = V\ W^*\ V'$ has to be minimized with respect to b. Notice that V = [V1 V2 V3 V4 V5 V6 V7 V8], and V_i (i=1, . . ., 8) is exactly as given in (3.24). That is, as in the single equation case, the second step is just a repetition of the first step, using W^* instead of W as the weighing matrix.

A grid search for values of b ranging from -50 to 50 yields a estimate of $\tilde{b}$ of -1.5681, with the quadratic form reaching a minimum value of .1983. In addition, when T times the minimum value of Q^* (which equals 6.3456) is compared with a chi-square random variable with 7 degrees of freedom, it is concluded that the relationships implied by both, the optimal path for the debt-to-asset ratio and consumption under constant relative risk aversion can not be shown to contradict aggregate reality, using Hansen's asymptotic test statistic.

Furthermore, the estimate of the asymptotic standard error of $\tilde{b}$ can be computed (see Chapter 2), letting

$$(3.26) \qquad D_{\delta t} = d[I(\delta_t)]/db = \sigma_{At} / (\hat{R}_{At} - K_t)$$

$$D_{Ct} = d[I(C_t)]/db = [(1/(1-b)^2)-(2b/(1-b)^3)]$$

$$[(\hat{R}_{At}-K_t)^2/2\sigma^2_{At}] + [K_t/(1-b)]-[(r_t-bK_t)/(1-b)^2]$$

where again, $I(\delta_t)$ refers to the implicit form of the optimal path equation for the leverage position, and $I(C_t)$ refers to the implicit form of the optimal path equation for consumption at time period t, and

$$D_1=\sum_{t=1}^{T}(D_{\delta t}\delta_t)/T \qquad D_2=\sum_{t=1}^{T}(D_{\delta t}K_t)/T$$

$$D_3=\sum_{t=1}^{T}(D_{\delta t}C_t)/T \qquad D_4=\sum_{t=1}^{T}(D_{\delta t}E_t)/T$$

$$D_5=\sum_{t=1}^{T}(D_{Ct}\delta_t)/T \qquad D_6=\sum_{t=1}^{T}(D_{Ct}K_t)/T$$

$$D_7=\sum_{t=1}^{T}(D_{Ct}C_t)/T \qquad D_8=\sum_{t=1}^{T}(D_{Ct}E_t)/T$$

Then, let $D = [D_1\ D_2\ D_3\ D_4\ D_5\ D_6\ D_7\ D_8]$ so that the asymptotic estimate of the standard error can then be computed as (since only one parameter is being estimated) $SE = (D\ W^*\ D')^{-1/2} = .4358$. In addition, since $\tilde{b}$ is strongly consistent and asymptotically normal, a 96% asymptotic confidence interval for b can be constructed by adding and subtracting twice the estimate of its asymptotic standard error to $\tilde{b}$. Therefore, the true underlying measure of risk aversion b, can

be expected (asymptotically) to be between -.6965 and -2.4397 with a 96% level of statistical confidence. Even if Tauchen's result regarding the possibility of $\tilde{b}$ being a biased estimator for b is considered, the relative risk aversion measure of the aggregate agricultural sector can be safely expected to be between zero and negative three. That is, by jointly considering the orthogonality conditions implied by both optimal paths, a much more reliable estimate of such measure is obtained. Empirically, a risk aversion coefficient within that range is associated with a moderately low level of risk aversion. Point estimates of aggregate relative risk aversion measures for the stock market, for example, range between -4 and -6. Alternatively, this result could be interpreted as suggesting that the inherent level of risk associated with participating in agricultural production is relatively high.

An alternative set of hypotheses regarding individual and aggregate behavior emerge under the assumption of constant absolute risk aversion. As in the case of constant relative risk aversion, limited (single equation) and full information (multiple equation) estimation based on these hypotheses can be attempted using the generalized method of moments approach. Since the mechanics of the generalized method of moments procedure have been extensively discussed both in general (chapter 3) and specific terms (under constant relative risk aversion), the methodology for estimation under constant absolute risk aversion will not be presented in detail. In

fact, the procedures are identical to those outlined for the case of constant relative risk aversion, and only the following definitions have to be altered:

$$(3.27) \qquad I(\delta) = \delta - 1 + (\lambda\ K\ \sigma_A^2\ E)\ /\ (\hat{R}_A - K))$$

$$I(C) = C - K\ E - \frac{[r - K + (\hat{R}_A - K)^2/2\sigma_A^2]}{\lambda\ K}$$

where λ is the absolute measure of risk aversion, and

$$d[I(\delta_t)]/d\lambda = K_t\ \sigma_{At}^2\ E_t\ /\ (\hat{R}_{At} - K_t)$$

$$d[I(C_t)]/d\lambda = \frac{[r_t - K_t + (\hat{R}_{At} - K_t)^2/2\sigma_{At}^2]}{\lambda^2\ K_t}$$

The limited information generalized method of moments estimate of λ based on the optimal path equation for the leverage position is $\tilde{b}$ = .000909. An estimate of the asymptotic standard error of $\tilde{b}$ is, SE = .003193, so that a 96% asymptotic confidence interval for b is given by $-.005477 < b < .007295$. Placing an absolute risk aversion measure within those boundaries has no practical significance, especially when the value of the outcome variable (aggregate consumption) is so large. As pointed out earlier, the larger the value of the outcome variable, the smaller the scale associated with the absolute risk aversion measure. The "optimal" quadratic form, however, achieves a minimum value of .1130. Therefore,

when T times the minimum value of the quadratic form associated with the most efficient estimate of b (which equals 3.616) is compared with a chi-square random variable with 3 degrees of freedom, it is concluded that the relationships implied by the optimal path for the debt-to-asset ratio under constant absolute risk aversion cannot be shown to contradict aggregate reality, using Hansen's asymptotic test.

The limited information generalized method of moments estimate of λ based on the optimal path equation for the consumption is $\tilde{b}$ = .15379. An estimate of the asymptotic standard error of $\tilde{b}$ is, SE = 4.3633, so that a 96% asymptotic confidence interval for b is given by -8.57281 < b < .8.88039. As pointed out earlier, placing an absolute risk aversion measure within those boundaries has no practical significance, specially when the value of the outcome variable (aggregate consumption) is so large. The "optimal" quadratic form, however, achieves a minimum value of .1217. Therefore, when T times the minimum value of the quadratic form associated with the most efficient estimate of b (which equals 3.8944) is compared with a chi-square random variable with 3 degrees of freedom, it is concluded that the relationships implied by the optimal consumption path under constant absolute risk aversion can not be shown to contradict aggregate reality either.

Finally, full information (multiple equation) generalized method of moments estimation under the assumption of constant absolute risk aversion can be attempted using both the optimal

path for the leverage position and consumption. The first-step quadratic form (based arbitrarily on a unit matrix), achieves a minimum value of zero at two different values of λ, so that two "optimal" weighing matrices become available. Using the first of those weighting matrices in the second step results in an estimate of λ, $\tilde{\lambda}_1$ = 19.9990, and the corresponding "optimal" quadratic form reaching a minimum value of .2156. An estimate for the asymptotic standard error of $\tilde{\lambda}_1$ is, SE_1 = 33.8731. Using the second weighing matrix results in an estimate $\tilde{\lambda}_2$ = -.02218, with the corresponding "optimal" quadratic form achieving a minimum value of .2117. An estimate for the asymptotic standard error of $\tilde{\lambda}_2$ is, SE_2 = .02359.

Several points regarding the results presented above, need to be addressed. First of all, the two second-step quadratic forms associated with the two "optimal" weighing matrices achieve almost identical minimum values. Under these circumstances it is harder to argue that the estimate associated with the quadratic form reaching the lowest value should be chosen. In practice, however, choosing $\tilde{\lambda}_2$ = -.02218 as the most efficient point estimate, and regarding $\tilde{\lambda}_1$ = 19.9990 as a less efficient estimate of the true underlying measure of absolute risk aversion implies no contradiction. That is, since the estimate of the asymptotic standard error of $\tilde{\lambda}_1$ is 33.8731, even a 68% asymptotic confidence interval for λ

constructed with the information regarding $\tilde{\lambda}_1$ and its asymptotic standard error, will contain $\tilde{\lambda}_2$.

Choosing $\tilde{\lambda}_2 = -.02218$ as the most efficient point estimate of λ, however, does not allow to place λ within meaningful bounds. A 96% asymptotic confidence interval for λ, in this case, is given by $-.06936 < \lambda < .02500$. Such confidence interval does not even allow to discriminate between risk averse and risk loving behavior. Even more disturbing is the fact that the most efficient point estimate of λ is negative (even to it is not asymptotically significantly different from zero). As mentioned earlier, a necessary condition for the given optimal paths to be considered a solution of the stochastic optimal control problem under constant absolute risk aversion is for λ to take only positive values (risk averse behavior).

Hansen's asymptotic test for the overidentifying restrictions jointly implied by the optimal paths for the debt-to-asset ratio and consumption under constant absolute risk aversion, however, does not allow to conclude that such restrictions are not consistent with aggregate behavior. That is, T times the minimum value of the quadratic form corresponding to $\tilde{\lambda}_2$ (which equals 6.7744) is a likely value for a chi-square random variable with 7 degrees of freedom. At this point it is important to keep in mind that there are no theoretical results or any other type of evidence (such as Monte Carlo simulations) regarding the validity of the power

of Hansen's asymptotic test for small samples. In other words, for the relatively small sample size of 32 used in this study, the aforementioned chi-square test may be considerably less powerful than the asymptotic results suggest, in which case the null hypothesis will be very difficult to reject if such asymptotic results are used to determine the rejection region for the test. In any event, even when jointly considering the orthogonality conditions implied by both optimal paths, it is not possible to place the absolute risk aversion measure between meaningful bounds. Given the scale in which the outcome variable (aggregate consumption) is being measured, the lower bound of the 96% asymptotic confidence interval $-.06936 < \lambda$ implies extreme risk loving behavior, while the upper bound $\lambda < .02500$ implies a high degree of risk adversity (Raskin and Lochran, 1986).

Even though Hansen's asymptotic test it is not able to reject the orthogonality conditions jointly implied the optimal paths for the debt-to-asset ratio and consumption, the negativity of the most efficient point estimate for λ, and the fact that the true underlying value of this risk aversion measure can not be placed within empirically meaningful bounds seem strong arguments against the validity of the set of hypotheses regarding individual and aggregate behavior emerging under the assumption of constant absolute risk aversion.

Aggregate Elasticities

Estimates of the effects of changes in exogenous variables such as the expected rate of return to assets, the variance of the rate of return to assets, the cost of debt capital, and the rate of inflation on aggregate debt level and consumption could be computed under both constant absolute and constant relative risk aversion. Since the optimal paths for the leverage position and consumption level under the assumption of constant absolute risk aversion can only be considered a solution of the stochastic optimal control problem for positive values of λ, and the full information generalized method of moments estimate of λ is negative, it would be theoretically unacceptable to conduct comparative statics analyses based on such optimal paths and the negative estimate of λ. Furthermore, even if negative values of the absolute risk aversion measure were theoretically acceptable, the estimate of the asymptotic standard error of the most efficient point estimate of λ is so relatively large that comparative statics results would be practically useless. For example, it would be impossible to determine within any reasonable degree of statistical certainty whether a change in any of the exogenous variables will result in an upward or a downward shift in the optimal paths. Therefore, it is decided not to conduct comparative statics analyses for the case of constant absolute risk aversion.

The hypotheses regarding aggregate behavior implied by the optimal path equations under constant relative risk aversion appear to be much more consistent with expectations. When the generalized method of moments technique based on the orthogonality conditions jointly implied by both optimal paths is applied, an estimate of the aggregate measure of relative risk aversion is obtained which is both very reasonable and relatively precise. The relatively small size of the estimate of the asymptotic standard error of $\tilde{b}$ will allow placing the various elasticity estimates between empirically relevant bounds.

The percentage change in the optimal debt-to-asset ratio, prompted by a one percent change in the expected rate of return to assets is

$$(3.28) \qquad (d\delta/d\hat{R}_A)/(\hat{R}_A/\delta) = \sigma_A^2\hat{R}_A(1-b)/\delta(\hat{R}_A-K)^2$$

Evaluating (3.28) at the mean values of the exogenous variables and $b = \tilde{b}$ yields an estimated elasticity of 1.4655. An estimate of the asymptotic standard error associated with this elasticity estimate can be easily computed (also at the mean values) if all of the exogenous variables entering (3.28), including the expected rate of return to assets, are assumed to be known constants. Such estimate is given by

$$(3.29) \qquad SE_{\delta,\hat{R}_A} = [\sigma_A^2\hat{R}_A/\delta(\hat{R}_A-K)^2]SE_{\tilde{b}} = .2486.$$

Since in reality the expected rate of return to assets is not known, an estimate of this variable has to be used in order to compute SE. Using an estimate of the expected rate of return to assets may result in (3.29) underestimating the asymptotic standard error of the elasticity estimate.

In addition, since $\tilde{b}$ is normally distributed (asymptotically), if all the exogenous variables can be assumed constant, the elasticity estimate can also be considered normally distributed. Under these circumstances a 96% asymptotic confidence interval can be constructed by subtracting and adding twice the estimate of its asymptotic standard error to the elasticity estimate. That is, if the expected rate of return to assets were to increase from its mean value of .0567 to .0624 (a 10% change), while all other exogenous variables remain at their mean values, the debt-to-asset ratio will be expected to increase from its mean value of .1387 to .1590. Furthermore, the new value will be expected to be between .1521 and .1659 with a 96% level of statistical confidence.

The percentage change in the optimal debt-to-asset ratio prompted by a one percent change in the cost of debt capital is given by

$$(3.30) \qquad (d\delta/dK)/(K/\delta) = -K\sigma_A^2(1-b)/\delta(\hat{R}_A-K)^2$$

Again, evaluating (3.30) at the mean values of the exogenous variables and $b = \tilde{b}$ yields an estimated elasticity of -.7032, and assuming that all exogenous variables are known

constants an estimate of the associated asymptotic standard error is given by

$$(3.31) \qquad SE_{\delta,K} = [K\sigma_A^2/\delta(\hat{R}_A-K)^2]SE_{\tilde{b}} = .1193.$$

A 96% asymptotic confidence interval for this elasticity estimate is then given by (-.9418, -.4646). That is, if the real cost of debt capital were to increase from its mean value of .0272 to .0299 (a 10% change), while all other exogenous variables remain at their mean values, the debt-to-asset ratio will be expected to decrease from its mean value of .1387 to .1289. Furthermore, the new value will be expected to be between .1256 and .1322 with a 96% level of statistical confidence.

The percentage change in the optimal debt-to-asset ratio prompted by a one percent change in the variance of the rate of return to assets is given by

$$(3.32) \qquad (d\delta/d\sigma_A^2)/(\sigma_A^2/\delta) = -\sigma_A^2(1-b)/\delta(\hat{R}_A-K)$$

Then, evaluating (3.32) at the mean values of the exogenous variables and $b = \tilde{b}$ yields an estimated elasticity of -.7622, and assuming that all the exogenous variables are known constants an estimate of the associated asymptotic standard error is given by

$$(3.33) \qquad SE_{\delta,\sigma_A^2} = [\sigma_A^2/\delta(\hat{R}_A-K)]SE_{\tilde{b}} = .1293$$

A 96% asymptotic confidence interval for this elasticity estimate is given by (-1.0209, -.5035). That is, if the variance of the rate of returns to assets were to increase from its mean value of .001214 to .001335 (a 10% change), while all other exogenous variables remain at their mean values, the debt-to-asset ratio will be expected to decrease from its mean value of .1387 to .1281. Furthermore, the new value will be expected to be between .1245 and .1317 with a 96% level of statistical confidence.

Comparative statics regarding the optimal path for consumption are not as simple. Changes in the exogenous paths will have an immediate direct effect on consumption through

$$(3.34) \qquad \frac{\{r(t)-K(t)b-(K(t)-\hat{R}_A(t))^2 b/2\sigma_A^2(t)(1-b)\}}{(1-b)}$$

which represents the optimal proportion of equity to be withdrawn from the farm operation over time.

The percentage change in the optimal consumption path, prompted by a one percent change in the expected rate of return to assets in the very short run (immediate and direct effect) is

$$(3.35) \qquad (dC/d\hat{R}_A)/(\hat{R}_A/C) = -Eb\hat{R}_A(\hat{R}_A-K)/C\sigma_A^2(1-b)^2$$

Evaluating (3.35) at the mean value of the exogenous variables and $b = \tilde{b}$ yields an estimated elasticity of 5.5312%. As in the case of the debt-to-asset ratio, an estimate of the

asymptotic standard error associated with this elasticity estimate can be computed (also at the mean values) if all of the exogenous variables entering (3.35), including the expected rate of return to assets, are assumed to be known constants. Since (3.35) is a nonlinear function of $\tilde{b}$, however, the following approximation has to be used (see Goldberger, 1964)

$$(3.36) \qquad \mathrm{Var}[F(\tilde{b},X)] = [dF(\tilde{b},X)/d\tilde{b}]' \, \mathrm{Var}[\tilde{b}] \, [dF(\tilde{b},X)/d\tilde{b}]$$

where $F(\tilde{b},X)$ is a non-linear function ((3.35) in this case) of the vector of parameter estimates $\tilde{b}$, and X (which represents the exogenous variables). Therefore, $dF(\tilde{b},X)/d\tilde{b}$ is, in general, a K by 1 vector of derivatives of $F(\tilde{b},X)$ taken with respect to each of the K parameters entering this function. $\mathrm{Var}[\tilde{b}]$ is an estimate of the covariance matrix of $\tilde{b}$. Following (3.36)

$$(3.37) \qquad SE_{C,\hat{R}_A} = -[(1+b)/(1-b)^3][E\hat{R}_A(\hat{R}_A-K)/C\sigma_A^2] \; SE_{\tilde{b}}$$

$$SE_{C,K} = [-EK/C(1-b)^2]+[(1+b)/(1-b)^3][EK(\hat{R}_A-K)/C\sigma_A^2]SE_{\tilde{b}}$$

$$SE_{C,\sigma_A^2} = [(1+b)/(1-b)^3][E(\hat{R}_A-K)^2/2C\sigma_A^2] \; SE_{\tilde{b}}$$

$$SE_{C,r} = Er/C(1-b)^2 \; SE_{\tilde{b}}$$

Using (3.37) to compute estimates of the standard errors associated with the elasticity estimates requires a number of

approximations. First of all, as in the case of the leverage position, an estimate of the expected rate of return to assets has to be used. Second, $SE_{\tilde{b}}$ is an estimate of the asymptotic standard error of $\tilde{b}$, so that it is only valid for large samples, strictly speaking. Third, since (3.36) is results from a Taylor series approximation of the true standard error of $F(\tilde{b},X)$, (3.37) will yield approximations of the true standard errors of the different non-linear functions of $\tilde{b}$. Therefore, if estimates of the standard errors are computed using (3.37), any inference based on such estimates (such as constructing confidence intervals for b), has to be looked upon with caution.

A second, indirect effect of a change in an exogenous variable on the optimal consumption path results from the effect of such change in the optimal evolution of equity over time. The expected total differential of equation (2.9), which describes the optimal evolution of equity over time is given by equation (3.9). Equation (3.9) implies the following

$$(3.38) \qquad dExp(dE^*)/d\hat{R}_A = 2E^*[1/(1-\delta^*)]$$

$$dExp(dE^*)/d\sigma_A^2 = (1-b)E^*[1/(1-\delta^*)]^2$$

$$dExp(dE^*)/dK = -2E^*[1/(1-\delta^*)]$$

That is, the firm will accumulate equity over time at a faster rate if the expected rate of return to assets increases. A decrease in either the variance of the rate of return

to assets or the cost of debt capital will also result in the firm accumulating equity at a faster rate. Therefore, an increase in the expected rate of return to assets will not only result in a higher proportion of the firm's equity being consumed every time period, but in higher consumption due to the firm holding relatively larger amounts of equity in the long run. The same type of argument applies to changes in the variance of the rate of return to assets and the cost of debt capital.

Because of the dynamic nature of this indirect effect of changes in the exogenous variables on the optimal consumption path (through equity accumulation over time), total elasticities capturing both, the direct and indirect effects, can not be computed. Simulation exercises such as the one performed by Ramirez, Moss and Boggess are recommended for analyzing the effect of changes in the exogenous variables on the optimal consumption path over time given particular scenarios that may be of interest.

CHAPTER 4
SUMMARY AND IMPLICATIONS

Agriculture in the United States suffered increased financial stress during the 1980s. Large debt loads incurred during the more prosperous 1970s turned to financial burdens as output and real estate prices declined. As a result, the agricultural sector underwent a period of foreclosures at the farm level and financial difficulties among agricultural intermediaries. The "debt is good" attitude of the 1970s was replaced by a more cautious approach.

The difficulties arising from the debt situation of the 1980s have spurred increased research into the appropriate level of farm debt and the factors influencing the farmer's leverage decision. The first part of this study attempts to deal with those issues within a comprehensive stochastic optimal control framework.

The individual farm owner is assumed to manage his operation according to the objective of maximizing a discounted infinite stream of expected utility. Utility is derived from consuming goods and services that can be purchased by withdrawing income from the firm. The first decision that the farmer has to face, therefore, is how much income to withdraw from the operation, every time period. This decision obviously affects the evolution of farm equity

value over time. The second decision involves the optimal leverage position at every time period.

The solution of stochastic optimal control problems, under the assumptions of constant relative and constant absolute risk aversion, yields two sets of optimal paths for the debt-to-asset ratio and consumption (income withdrawal) over time. Analysis of these optimal paths reveals that important characteristics individual behavior depend on whether constant relative or constant absolute risk aversion is assumed.

For example, under constant relative risk aversion the optimal leverage position does not depend on the firm's equity holdings while under constant absolute risk aversion it is a decreasing function of the amount of equity held by the firm. For the aggregate agricultural sector, where larger farms has been a long-run trend, constant absolute risk aversion would imply a decreasing leverage position over time, on the average. Constant relative risk aversion on the other hand, would imply a stationary aggregate leverage position, if all the exogenous paths also affecting the farm level optimal debt-to-asset ratio (i.e. the expected rate of return to assets, variance of the rate of return to assets and cost of debt capital) are assumed stationary.

Furthermore, the optimal average propensity to consume (out of disposable income) is also independent of equity value under constant relative risk aversion. Under constant

absolute risk aversion, however, as the firm accumulates equity over time, the optimal average propensity to withdraw income from the farm operation decreases. As in the case of the leverage position, farm level choices should be reflected on the aggregate behavior of the agricultural sector. As the typical farm becomes larger, and the typical farm owner finds himself holding more and more equity, an aggregate trend to withdraw lower proportions of the sector net returns should be observed, if farmers' behavior were consistent with constant absolute risk aversion. That is, the sector would be more able (and willing) to finance its growth with internal revenues.

The second part of the study deals with estimating risk aversion measures, and testing the restrictions implied by the optimal path equations under the assumptions of constant absolute and constant relative risk aversion, for the aggregate agricultural sector. Even though the restrictions implied by the assumption of constant absolute risk aversion can not be rejected using the only (asymptotic) test available, it is concluded that the associated optimal paths do not describe the behavior of the aggregate agricultural sector very well. In any event, since the risk aversion measure can not be placed within empirically meaningful bounds under constant absolute risk aversion, if the corresponding optimal paths were used in either simulation or comparative statics

analysis, the results would lack of any credibility from a statistical standpoint.

The optimal paths associated with the assumption of constant relative risk aversion, on the other hand, appear to be quite consistent with the aggregate behavior of the agricultural sector. Furthermore, the risk aversion measure can be placed within empirically meaningful bounds in this case. Results from comparative statics analysis investigating the effect of changes in exogenous paths (such as the expected rate of return to assets, variance of the rate of return to assets, cost of debt capital, and inflation rate), on the optimal leverage position and consumption, prove to be quite reliable, from a statistical perspective.

APPENDIX
THE SOLUTIONS OF THE STOCHASTIC OPTIMAL CONTROL PROBLEMS

Constant Relative Risk Aversion

The stochastic optimal control problem is to maximize

$$\text{Max Exp } \left(\int_0^{\infty} e^{-rt} \, (C(t)^b)/b \;\; dt \right)$$

Subject to

$$dE(t) = \left[E(t) \; [\hat{R}_A(t) - K(t)\delta(t)] \, \frac{1}{(1 - \delta(t))} - C(t) \right] dt$$

$$+ \; E(t) \, \sigma_A(t) \, \frac{1}{(1 - \delta(t))} \, dz(t)$$

and $E(0) = E_0$.

The corresponding basic equation is

$$r(t)V[E(t)] = \max_{C,w} \; \{ C^b(t)/b + V'[E(t)]\{[E(t)[\hat{R}_A(t)$$

$$-K(t)\delta(t)]/(1-\delta(t))]-C(t)\} + .5w^2(t)E^2(t)\sigma_A^2(t)V''[E(t)]\}$$

In order to make the basic equation a more linear function of the control variables, let $w(t) = 1/(1-\delta(t))$. The newly defined control variable, $w(t)$, can be easily shown to equal the assets to equity ratio. The transformed basic equation is[2]

$$r(t)V[E(t)] = \max_{C,w} \{ C^b(t)/b + V'[E(t)]\{E(t)[\hat{R}_A(t)w(t) + K(t)(1-w(t))]-C(t)\} + .5w^2(t)E^2(t)\sigma_A^2(t)V''[E(t)]\}$$

The maximizing values of $C(t)$ and $w(t)$ in terms of the parameters of the problem can be obtained by taking the corresponding partial derivatives, setting them equal to zero, and solving,

$$C^*(t) = \{V'[E(t)]\}^{1/(1-b)}$$

$$w^*(t) = V'[E(t)](K(t)-\hat{R}_A(t))/\sigma_A^2(t)E(t)V''[E(t)]$$

Substituting in $C^*(t)$ and $w^*(t)$ for $C(t)$ and $w(t)$ respectively, and simplifying, yields the following non-linear second order differential equation

$$r(t)V[E(t)] - (1-b)\{V'[E(t)]\}^{b/(1-b)}/b - K(t)E(t)V'[E(t)] + (\hat{R}_A-K(t))^2\{V'[E(t)]\}^2/2\sigma_A^2(t)\{V''[E(t)]\} = 0$$

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[2] The transformed version of this problem has already been solved by Merton (1969).

which can be shown to hold if $V[E(t)] = AE(t)^b$, where A is some positive function of the parameters of the problem. Computing $V'[E(t)]$ and $V''[E(t)]$, substituting the results in the differential equation, and simplifying yields

$$Ab = \left[\left[r - K(t)b - \frac{(K(t) - \hat{R}_A(t))^2 b}{2\sigma_A^2(t)(1-b)} \right] \frac{1}{1-b} \right]^{b-1}$$

Since A proves not to be a function of E(t), $V[E(t)] = AE(t)^b$ is indeed a solution for the non-linear second order differential equation under consideration. The optimal paths (equations (5.1) and (5.2)), therefore, can be obtained by substituting the appropriate derivatives into $C^*(t)$ and $\delta^*(t) = 1 - 1/w^*(t)$.

<u>Constant Absolute Risk Aversion</u>

The stochastic optimal control problem is to maximize

$$\text{Max Exp } \left(\int_0^\infty e^{-rt} \, (-e^{-\lambda C(t)})/\lambda \quad dt \right)$$

Subject to

$$dE(t) = \left[E(t) \, [\hat{R}_A(t) - K(t)\delta(t)] \frac{1}{(1 - \delta(t))} - C(t) \right] dt$$

$$+ \; E(t) \, \sigma_A(t) \frac{1}{(1 - \delta(t))} dz(t)$$

and $E(0) = E_0$.

The corresponding basic equation is

$$r(t)V[E(t)] = \max_{C,w} \{ (-e^{-\lambda C(t)})/\lambda + V'[E(t)]\{[E(t)[\hat{R}_A(t)$$

$$-K(t)\delta(t)]/(1-\delta(t))]-C(t)\} + .5w^2(t)E^2(t)\sigma_A^2(t)V''[E(t)]\}$$

In order to make the basic equation a more linear function of the control variables, let $w(t) = 1/(1-\delta(t))$. The newly defined control variable, $w(t)$, can be easily shown to equal the assets to equity ratio. The transformed basic equation is[3]

$$r(t)V[E(t)] = \max_{C,w} \{ (-e^{-\lambda C(t)})/\lambda + V'[E(t)]\{E(t)[\hat{R}_A(t)w(t)$$

$$+K(t)(1-w(t))]-C(t)\} + .5w^2(t)E^2(t)\sigma_A^2(t)V''[E(t)]\}$$

The maximizing values of $C(t)$ and $w(t)$ in terms of the parameters of the problem can be obtained by taking the corresponding partial derivatives, setting them equal to zero, and solving

$$C^*(t) = -\ln\{V'[E(t)]\}/\lambda$$

$$w^*(t) = V'[E(t)](K(t)-\hat{R}_A(t))/\sigma_A^2(t)E(t)V''[E(t)]$$

[3] The transformed version of this problem has already been solved by Merton (1969).

Substituting in $C^*(t)$ and $w^*(t)$ for $C(t)$ and $w(t)$ respectively, and simplifying, yields the following non-linear second order differential equation

$$V'[E(t)]E(t)K(t) + V'[E(t)]\ln\{V'[E(t)]\}/\delta - r(t)V[E(t)]$$
$$- V'[E(t)]/\delta - (\hat{R}_A - K(t))^2\{V'[E(t)]\}^2/2\sigma_A^2(t)\{V''[E(t)]\} = 0$$

which can be shown to hold if $V[E(t)] = -(P/Q)\exp^{-QE(t)}$, where P and Q are functions of the parameters of the problem. After computing $V'[E(t)]$ and $V''[E(t)]$, substituting the results in the differential equation, and simplifying, it is found that P and Q must satisfy the following equations

$$P = \exp\{[K(t) - r(t) - (\hat{R}_A(t) - K(t))^2/2\sigma_A^2]/K(t)\}$$
$$Q = \lambda K(t)$$

Since P and Q prove not to be functions of $E(t)$, $V[E(t)] = -(P/Q)\exp^{-QE(t)}$ is indeed a solution for the non-linear second order differential equation under consideration. The optimal paths (equations (5.11) and (5.12)), therefore, can be obtained by substituting the appropriate derivatives into $C^*(t)$ and $\delta^*(t) = 1 - 1/w^*(t)$.

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BIOGRAPHICAL SKETCH

Octavio Alberto Ramirez was born on October 16, 1963, in Managua, the capital city of the Central American country of Nicaragua. He attended high school at "Colegio Centro-america," a Catholic institution, and was graduated in December of 1981.

The political and economic instability of his native country, at that time, forced him to seek opportunities for higher education outside of Nicaragua. He and his family decided that it was best to accept a scholarship from the Panamerican Agricultural School (El Zamorano), a highly selective institution located in the neighboring country of Honduras. In December of 1984, he received a technical degree in agronomy from the Panamerican Agricultural School and, as the valedictorian of a class of over one hundred students representing most Latin American countries, was awarded the "Wilson Popenoe" scholarship.

After a year of working for the Integrated Pest Management Project of El Zamorano, Octavio enrolled in the University of Florida at Gainesville, to pursue a bachelor's degree in agricultural economics. In January of 1987, he started working towards a master's degree in the Food and Resource Economics Department, also at the University of Florida. In

December of 1987, he received his Master of Science degree and married Renelle, a Gainesville native. He also decided to continue on for a doctorate in agricultural economics at the University of Florida. His longstanding fascination and good handling of mathematics and statistics made the choice of econometrics as his field of specialization, an easy one. While attending the University of Florida, Octavio enjoyed the generous financial support of the Food and Resource Economics Department, where he was employed as a research assistant. He also maintained a 4.00 grade point average throughout his undergraduate and graduate studies.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

John Scott Shonkwiler, Chairman
Professor of Food and Resource Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Robert D. Emerson
Professor of Food and Resource Economics

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Timothy G. Taylor
Associate Professor of Food and Resource Economics

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Charles B. Moss
Assistant Professor of Food and Resource Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

G. S. Maddala
Graduate Research Professor of Economics

This dissertation was submitted to the Graduate Faculty of the College of Agriculture and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August, 1990

Jack L. Fry
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Dean, Graduate School